1. Introduction : (b) Pareto Optimality in an Exchange Economy

As I warned earlier, AP/ECON 4070 is a course in applied microeconomic theory. So the main part of this introductory section is a review of the important microeconomic concepts which will be used frequently in the course. Ideally, this material should be familiar to you from AP/ECON 2300 & 2350, and the material is certainly covered in great detail in any intermediate micro textbook, for example *Varian : Intermediate Microeconomics* or *Nicholson : Microeconomic Theory.* [Probably the best intermediate micro textbook available these days is *Introduction to Economic Analysis* by McAfee and Lewis : it's actually free to read online, but ebook and printed versions are available for a relatively low price ; see `http://www.introecon.com`].

Begin with an economy in which there are no economic institutions : no prices, no firms, no markets, no government. In fact, to make life simpler, in this imaginary economy there is no production of goods and services. So what economic activity is there, if there is no production, no prices, no government....? All there are are **endowments** of different commodities. So this "pre–institutional" economy consists of a bunch of people, and fixed quantities of a bunch of commodities. In this very simple economy, there is only one economic question : how to divide the given quantities of the different commodities among the different people. Even though this is an extremely artificial and simple world, it turns out that solving this problem says a lot about economic policy in a lot of (more realistic) settings.

In other words : how to divide up a fixed stock of a bunch of different goods and services among a group of people is a very important economic question.

To make life as simple as possible, assume that there are just 2 people, and just 2 commodities to divide between them. Remember, there is no production being done here : we have X units of one good, and Y units of another good, and these quantities are fixed. (An economy in which there is no production, only exchange of a given stock of goods and services is usually called, not surprisingly, an **exchange economy**.) These commodities are "goods" : each person wants more of each commodity. The only economic question in this economy is how to **allocate** the fixed endowments X and Y of the two goods among the two people. What must be chosen is an allocation. In this very simple world, with two people and two goods, an allocation is simply four numbers : x_1, x_2, y_1 and y_2. Here the two people are named "1" and "2", so that x_2, for example, is the allocation of good X to person 2. An allocation is **feasible** if it does not give out more of the commodities than are available. That is, the allocation (x_1, x_2, y_1, y_2) is feasible if

$$x_1 + x_2 \leq X$$

, and if

$$y_1 + y_2 \leq Y$$

The first inequality says that the quantity of food (x_1) given to person 1, added to the quantity of food (x_2) given to person 2, cannot be more than the fixed total quantity of food X, which is available for distribution to the two people. More generally, if there were 100 people, and 26 different commodities $A, B, C, \ldots, Z$, then an allocation would be a list of 2600 numbers, each representing a quantity of a different commodity to a different person, so that m_{23} would be the quantity of good M going to person # 23. In this 26–by–100 economy the conditions for feasibility would be

$$a_1 + a_2 + a_3 + \cdots + a_{100} \leq A$$

$$b_1 + b_2 + b_3 + \cdots + b_{100} \leq B$$

and so on, up to

$$z_1 + z_2 + z_3 + \cdots + z_{100} \leq Z$$

where A and B are the total quantities available of the first two commodities, and Z is the quantity available of the 26th commodity.

With two people and two goods, there is a fairly convenient way to represent allocations graphically. That is the (familiar, I hope) Edgeworth box diagram. The trick in using an Edgeworth box is to put all 4 numbers : x_1, x_2, y_1, y_2 : into one picture.

That trick is achieved by the following piece of logic : These commodities are goods, not bads. So the more each person gets, the happier she or he is. That being the case, we want to use the goods to the fullest. That means that we really should be most interested in allocations in which we allocate all the available goods, those for which

$$x_1 + x_2 = X$$

and

$$y_1 + y_2 = Y$$

If an allocation were feasible, but did not give out all of the stock of the commodities (that is if $x_1 + x_2 < X$, or if $y_1 + y_2 < Y$), then we could make one person better off, without making the other person worse off, simply by using some of the available stock which is left over. And making people well off is — by assumption — the name of the game here.

In a two–person, two–good economy, any food which doesn't go to person #1 or to person #2 is wasted — there's nobody else. So if we are interested in good allocations, allocations which people like, we should concentrate on allocations which don't waste any of the food or clothing.

So, in using the Edgeworth box diagram, we consider only allocations which allocate *all* of the available quantities of the commodities. That means that anything person #1 does not get, will go to person #2. That trick enables the construction of the Edgeworth box. The box is a rectangle ; its dimensions are the quantities of the 2 goods. So it's X wide and Y high.

Figure 1 depicts an Edgeworth box for a 2–person exchange economy, in which the total quantity X of food is 12 units, and in which the total quantity Y of clothing is 16 units.

Person #1's consumption is measured from the *bottom left* corner of the box. So suppose some allocation gives person #1 x_1 kilos of food, and y_1 metres of cloth. That means we measure x_1

units to the right of the bottom left corner of the box, and y_1 units up. That measurement leads to some point in the box.

How far is that point from the right side of the box? Person #1 has x_1 kilos of food, and there are X kilos of food in total. So the point is $X - x_1$ from the right side of the box. Similarly, the point is $Y - y_1$ from the top edge of the box. And those quantities, $X - x_1$, and $Y - y_1$, will be the quantities of food and cloth allocated to the other person, person #2.

Figure 2 depicts an allocation in the Edgeworth box, one in which $x_1 = 8$, and $y_1 = 4$ (and in which $X = 12$ and $Y = 16$, just as in figure 1). That means that person 2 gets $12 - 8 = 4$ units of food, and $16 - 4 = 12$ units of cloth.

In other words, any point in the box represents an allocation — and any allocation can be represented by a point in the box. At any point, the distance from the left edge represents the quantity of food allocated to person #1, and the distance from the right edge the quantity of food allocated to person #2. The distance from the top edge is the quantity of clothing allocated to person #2, and the distance from the bottom edge is the quantity of cloth y_1 allocated to person #1. This means that the Edgeworth box is measuring person #1's consumption from the bottom left corner, and person #2's from the top right corner. Moving down the box vertically, for example, means allocating more cloth to person #2 and less to person #1.

Next, how the two people feel about different allocations can be represented. Each person is assumed to care only about what she or he gets, and to want more of each good. That means that I can draw indifference curves for each person. These indifference curves slope down for person #1, indicating that if she is indifferent between two consumption bundles (x_1, y_1) and (x_1', y_1'), and $x_1 < x_1'$, then it must be the case that $y_1' < y_1$. If we give her more food in the second bundle, and if she is indifferent between the two bundles, then we must be giving her less cloth in the second bundle.

Person #2's indifference curves slope down as well. If we give him less food in one bundle, we must give him more cloth to keep him on the same indifference curve. So $x_2' < x_2$ means $y_2' > y_2$ if (x_2, y_2) and (x_2', y_2') are on the same indifference curve for person #2. That says that if the second bundle is to the right of the first bundle (lower food consumption for person #2), then it must be below the first bundle (more cloth consumption) if person #2 is indifferent between the two bundles.

I will also draw the indifference curves representing **convex** preferences. That means each person's indifference curves are "bowed in" towards their respective origins. A person has convex preferences if : whenever she likes two bundles just as much as each other, then she likes even more the bundle which is "between" the first two bundles. If I like the bundle $(10, 1)$ containing lots of food and little cloth just as much as the bundle $(3, 15)$ containing mostly cloth, then — if I have convex preferences — I must like even better a combination of the two bundles, such as the bundle $(7, 7)$, which is on the line connecting the first two bundles.

DEFINITION : Person 1's preferences are **convex** if, whenever there are two bundles (x_1, y_1)

and (x'_1, y'_1) which are on the same indifference curve for her, then any bundle (x''_1, y''_1) which is on a line connnecting the first two bundles (x_1, y_1) and (x'_1, y'_1) will be on at least as high an indifference curve for her as the bundles (x_1, y_1) and (x'_1, y'_1).

The bundle (x''_1, y''_1) is on a line connecting (x_1, y_1) and (x'_1, y'_1) if and only if there is some fraction t, between 0 and 1, such that

$$x''_1 = tx_1 + (1-t)x'_1$$

$$y''_1 = ty_1 + (1-t)y'_1$$

For example, if $(x_1, y_1) = (3, 10)$, and if $(x'_1, y'_1) = (15, 4)$, then $(7, 8)$ is on the line connecting these two points, since $7 = (2/3)3 + (1/3)15$, and $8 = (2/3)10 + (1/3)4$: here $t = 2/3$, and $(7, 8)$ is one-third of the way along a line connecting $(3, 10)$ with $(15, 4)$.

So convex preferences say that if a person was indifferent between the bundles $(3, 10)$ and $(15, 4)$, then she would find bundles such as $(6, 8.5)$, $(7, 8)$, $(9, 7)$ and $(13, 5)$ all to be at least as good as the original bundles $(3, 10)$ and $(15, 4)$

Convexity of preferences is an important property in much of the course. It's a property we simply assume — it's relatively easy to think of little examples in which people might not feel this way. But I will assume throughout the course that people have convex preferences. What that means is that if you take any two bundles on an indifference curve, then all the bundles on the line connecting these two bundles are on higher indifference curves. When there are only two commodities (and when we are measuring consumption from the bottom left), then convexity means that we have a diminishing marginal rate of substitution — the indifference curves get less steep as we move down and to the right.

Since we are measuring the consumption of person #2 from the top right corner of the Edgeworth box, his indifference curves are "mirror images" of those of person #1. They will get steeper as we move down and to the right if his preferences are convex. The reason? For him the direction of higher indifference curves is down and to the left : southwest. You should be able to check that if the line connecting two points on one of his indifference curves is on a higher indifference curve, then the indifference curve has to get steeper as we move down and to the right.

Each point in the Edgeworth box corresponds to an allocation. What are the good allocations? To answer that, consider instead what are the bad allocations. Take some allocation in the Edgeworth Box. Draw through it the indifference curves of each of the people. Person #1 prefers, to this allocation, every allocation which is on a higher indifference curve for her — those allocations are to the northeast of her indifference curve through the allocation. Person #2 prefers every allocation which is on a higher indifference curve for him — those allocations on his indifference curves to the southwest. An allocation B is described as **Pareto preferred** to some allocation A if both person #1 and person #2 prefer B to A. [1]

[1] Vilfredo Pareto was an Italian/Swiss economist/sociologist of the 19th/20th centuries. The terms "Pareto preferred", "Pareto optimal", and "Pareto efficient" are named after him.

In Figure 3, the two people have convex preferences, since person 1's indifference curve gets less steep as we move down and to the right, and person 2's get more steep as we move down and to the right. In Figure 3, the allocation B is Pareto preferred to the allocation A. Person 1 likes B better, because B is northeast of her (solid, red) indifference curve through A ; person 2 likes it better because B is to the southwest of his (dotted, green) indifference curve through A.

In fact, whenever the indifference curves of the two people through A have different slopes, then there will be other allocations which are Pareto–preferred to A. If the indifference curve of person #1 is steeper, as in Figure 3, then there will be a lens–shaped area to the southeast of A which both people prefer. How can both people be made better off, with the same total quantity of each commodity? In this example, trade one good for the other. Moving southeast from A to B involves person #1 giving up some of her cloth, in exchange for more food. Person #2, then, is giving up food to get cloth. If person #1 has a strong taste for food — compared to person #2 — this is a deal which can make both people better off. On the other hand, if the indifference curve of person #2 is steeper, then a Pareto–improvement can be implemented by making a deal in the opposite direction, to the northwest.

So an allocation is "bad" if I can find a Pareto improvement from it, since that way I can make both people better off. Pareto preferred means exactly : preferred by all people. (Or, more precisely : at least as good for all people, and strictly better for some people.)

The concept of a "good" allocation is then simple enough : an allocation is **efficient**, or **Pareto optimal**, if it is **not** possible to find a Pareto improvement from it. An allocation will be Pareto optimal if the indifference curves through it of the two people have the same slopes — that is, if they are tangent. If the indifference curves through an allocation are tangent, then allocations with person #1 likes better than the allocation, are allocations which person #2 likes worse.

There is a whole family of Pareto optimal allocations. They form a curve in the Edgeworth box, called the **contract curve**. Each allocation on the contract curve has the indifference curves through it of the two people tangent. Each allocation on the contract curve is "good", in the sense that you cannot make one person better off from such an allocation, without making the other person worse off.

Figure 4 shows a contract curve (the solid red line), and two indifference curves for each of the two people. At every point on the contract curve (not just the two points illustrated in the figure), the indifference curve of person 1 through the allocation is tangent to the indifference curve of person 2. (So person 1's dark blue indifference curve is tangent to person 1's light blue indifference curve at a point on the solid red contract curve, in Figure 4, for example.) Note that in this figure, the contact curve slopes up. Moving up the contract curve means making person #1 better off, and person #2 worse off. The two corners (bottom left and top right) of the Edgeworth Box are Pareto optimal, if both commodities are "goods". The bottom left corner is the allocation in which $x_1 = 0$, $x_2 = X$, $y_1 = 0$, $y_2 = Y$. In other words, person #2 gets everything. That allocation is Pareto optimal, since there is no other allocation preferred by both people. Sure, there are lots of allocations which person #1 likes better. But there is no allocation which person #2 likes better, and since we can't make him better off, then we cannot find a Pareto improvement.

In the world of the Edgeworth box, there is a conflict along the contract curve, between person 1's well–being and person 2's. Economic theory does not say much about how to resolve this conflict. But there really is not a conflict here between equity and efficiency. Soon, when more realistic complications are introduced, we will face a trade–off between the principles of efficiency and equity. But in the **first–best** world that I have been describing so far, depicted in the Edgeworth box, there are an infinite number of efficient allocations : all the points on the contract curve. Some of these allocations may seem very inequitable, such as allocations giving nearly everything to person #1. But they are all **efficient**, since there are no trades possible which make both people better off.

On the other hand, any allocation off the contract curve is inefficient. Even if an allocation off of the contract curve looks somehow equitable, it's still not a very good allocation, because we could make both people even better off still, by trading the good that person #1 likes least to person #2, in exchange for some of the good that person #1 likes relatively more.

Now, before introducing production of goods and services into this world, something should be said about prices and markets. Remember, the world of the Edgeworth box, so far, is not a world of prices and markets. It is a world in which some dictator allocates food and clothing to people.

The market system is a mechanism for allocating goods and services. It is certainly not the only mechanism. Perfectly competitive markets are markets in which all people, buyers and sellers, face the same prices for goods and services. The fact that all consumers in competitive markets pay the same price for food is worth emphasizing. Later, introducing taxes will change that fact. These people in a competitive economy take the prices as given. So each person, given the prices of goods and services, chooses the consumption bundle which she prefers most, from the ones she can afford. The consumption bundles she can afford, I hope you recall, are those in her budget set. So if the price of food were p_x and the price of clothing were p_y, then person 1 could afford any bundle (x, y) for which $p_x x + p_y y \leq M_1$, if M_1 is her income.

The bundles that she just can afford are arranged on a line, with slope $-p_x/p_y$, if her food consumption is graphed on the horizontal, and her clothing consumption on the vertical. In a competitive economy, she chooses the bundle on that line which is on the highest of her indifference curves. That is, she finds the bundle on the budget line where the indifference curve through the bundle is tangent to the budget line, that is where the slope of her indifference curve $MRS_1 \equiv MU_x^1/MU_y^1$ equals the slope p_x/p_y of the budget line. This tangency of the indifference curve with the budget line determines the person's demands for different goods and services in a perfectly competitive market economy.

This all should be very familiar to you. The one wrinkle I want to throw in here is the determination of a person's income here. There's no production yet in my simple Edgeworth box economy, and no government. So where does the person's income come from? The way income is determined here is to assume that all of the food and clothing in the economy is owned by someone.

Person 1 is **endowed** with ownership of food and cloth. She gets income by selling some of her endowment — at market prices p_x and p_y. So she will either sell some of her endowment of cloth to buy food, or sell some of her endowment of food to buy cloth, depending on the prices and on her tastes. If $\bar{x_1}$ denotes person 1's endowment of food, then the total value of her endowment is $M_1 = p_x\bar{x_1} + p_y\bar{y_1}$. That's what she would get if she sold off all the food and cloth that she owned. The equation of her budget line then becomes

$$p_x x + p_y y \leq p_x\bar{x_1} + p_y\bar{y_1}$$

which also can be written

$$p_x(x - \bar{x_1}) = -p_y(y - \bar{y_1})$$

That is, the cost of her net purchases of food must equal the value of her net sales of cloth.

The endowments of food and cloth are starting points here. I don't explain where they came from. They must have been determined through some history or culture. This ownership pattern certainly influences what happens in a market economy. But it's taken as given. All the food and cloth in the economy is owned by someone. So if there were only two people in this economy, then it would have to be the case that

$$\bar{x_1} + \bar{x_2} = X \quad ; \quad \bar{y_1} + \bar{y_2} = Y$$

That means, in the two person economy, I can use the Edgeworth box to depict endowments of goods and services. If person 1's endowment $(\bar{x_1}, \bar{y_1})$ is measured from the bottom left of the box, then person 2 is endowed with whatever person 1 does not own. His endowment can be measured from the top right. An endowment point near the bottom right of the box, for example, means that person 1 owns most of the food and person 2 owns most of the cloth.

They don't have to consume their endowments in a market economy. They are free to buy and sell them, and each person chooses a consumption point which is the point she prefers most of all the ones she can afford. The points person #1 can afford are the ones along her budget line. Her budget line is a line with slope $-p_x/p_y$. Which line? Her budget line must go through her endowment point, since that's a point she can exactly afford. If she wants to consume exactly $\bar{x_1}$ units of food, then the most cloth she can consume is $\bar{y_1}$: if she doesn't sell any of her endowment of food, she cannot buy any more cloth.

So the whole market process, with price–taking consumers, can be represented in my Edgeworth box. The endowment point is some point in the box. One line, through this endowment point, with slope $-p_x/p_y$, represents both people's budget lines. Given the endowment point and the prices, each person wants to consume at a point on this line where her or his indifference curve is tangent to the budget line.

Which leads to the question, how are the prices determined? Are the prices p_x and p_y arbitrary, just like the people's endowments? **No**. In a market economy, prices must clear the markets. The quantity demanded of food must equal the quantity supplied. That is,

$$x_1 + x_2 = \bar{x_1} + \bar{x_2}$$

$$y_1 + y_2 = \bar{y_1} + \bar{y_2}$$

If person 1 wants to consume more cloth than she owns, this cloth must come from person 2.

So prices (p_x, p_y) are **equilibrium prices**, given the endowments of the goods and services, and given the people's tastes, if all markets clear. Two alternative ways of saying that are : if the total quantity demanded of each good equals the total quantity available, or if one person's net sales of any good equals the other person's net purchases.

Graphically, what that means is that person 1's preferred point must coincide, in the Edgeworth box, with person 2's. Only if the two points coincide will markets clear. Given the endowments $\bar{x_1}, \bar{y_1}, \bar{x_2}, \bar{y_2}$, and given people's tastes, the prices p_x, p_y are equilibrium prices if markets clear, and the corresponding demands x_1, y_1, x_2, y_2 of the two people are the equilibrium allocation of goods and services.

So two distinct mechanisms for allocating goods and services have been described in this note. The first was to have some dictator allocate the goods and services so that there was no way of making one person better off without making the other person worse off. That mechanism led to a bunch of allocations, all on the contract curve. The second was the price mechanism. Here we start with a given ownership pattern, and ask what prices will clear the markets, given the ownership pattern. The quantities demanded by the people, at these equilibrium prices, are the equilibrium allocation. if the economy had a different ownership pattern, then there would be a different equilibrium price vector (probably), and a different equilibrium allocation (probably).

The central results in microeconomics are probably that these two mechanisms tend to give rise to the same allocations. This relation is usually given by two fundamental theorems. The First Fundamental Theorem is the easier one. It says the following. Start with an endowment pattern ; find the equilibrium prices and then the equilibrium allocation ; that equilibrium allocation will be on the contract curve. In other words, every competitive equilibrium is Pareto efficient. The Second Fundamental Theorem reverses direction. It says the following. Start with an efficient allocation, that is one on the contract curve. Then there is some endowment $\bar{x_1}, \bar{y_1}, \bar{x_2}, \bar{y_2}$ which has that efficient allocation as its competitive equilibrium. In other words, every Pareto efficient allocation can be sustained as a competitive equilibrium, if you reallocate ownership of goods and services right.

The second fundamental theorem, in particular, has very strong implications for the economics of the public sector. It says that the government doesn't have to do very much. It seems reasonable – I hope – that if we got to pick an allocation, we should pick an efficient one. The second theorem says that if you want to allocate goods and services efficiently, then private markets will do it, provided that they're competitive, and provided that the ownership of goods and services is distributed right.

The proofs of these results are fairly simple, at least in my two–person, two–good world with no production. Start with a competitive equilibrium. At any competitive equilibrium, person 1's indifferent curve must be tangent to the same budget line as person 2's, at the same point. That means that $MRS_1 = p_x/p_y = MRS_2$, so that $MRS_1 = MRS_2$, which was the condition for efficiency. The second fundamental theorem? Start at any point A on the contract curve. At that point, $MRS_1 = MRS_2$. So the two people's indifference curves have the same slope, call it s. Take a line through the point on the contract curve, with that slope s. Both people's indifference curves will be tangent to this line, at this efficient point. So if the people's endowment point were any point on this line, then there would be a competitive equilibrium, with $p_x/p_y = s$, at the point A.

These results do not depend on there being only two people, or only two goods, or on there being no production. An efficient allocation of goods and services requires that for any two goods, and any two people, that one person's MRS of one good for the other equals the other person's MRS. if both people face the same prices, then each person will choose to consume where her MRS equals the price ratio, so that competitive equilibrium also gives rise to the equality of MRS's.