1. Introduction : (e) Consumer's Surplus

Another concept from intermediate microeconomics which gets used in public economics is the concept of **consumer's surplus**. Note that this is a singular "consumer's" ; I want to look here at the surplus **one** consumer gets. A setting in which consumer's surplus is used a lot, in public economics, is measuring the impact of taxation. One of the main topics covered in this course is measuring the cost of a given tax to a given consumer. That is, "what is the cost, in dollars per year, to a particular consumer of there being a tax of $2 per pack on cigarettes?". A closely related question : "how would that cost change with a change in the tax?".

Here is a simple, obvious — but not quite correct — answer. The tax is $2 per pack. The person smokes 500 packs per year. Therefore, the cost to her of the tax would be $2 \cdot 500 = 1000$ dollars per year.

That answer is not quite correct. Why not? The person's cigarette consumption is not **fixed** at 500 packs per year. It's a quantity that she **chooses**. How much consumers buy of different goods and services is a function of the prices of the different goods and services (and of their incomes, and of their tastes). So putting a $2 per pack tax on cigarettes might change her consumption from 500 to 400 packs per year. So what is the cost to her of the tax, $800 or $1000? The correct answers will be between $800 and $1000. Notice that it's "answers", in the plural, in that last sentence. In general, there is more than one measure of consumer's surplus.

Two hypothetical questions : First, "how much would this consumer be willing to bribe me not to introduce this tax?". Second, "how much would I have to compensate the consumer for the damage done to her by the tax?". The answers to these questions are both measures of the cost to this consumer of the tax. The bad news is that those are two different questions, with answers that will, in general, differ.

To see why, perhaps, consider a person earning $30,000 per year in some job. Suppose that, instead of looking at the cost of a tax, I was looking at the dollar cost to the person of working in a very dangerous job, where there is a serious threat of major injury or death. The first question I could ask is "what would this person be willing to bribe me in order to ensure that she was not exposed to such danger on the job?". The second, "what would I have to compensate her for the damage done by having a dangerous job?". We might expect the answer to the second question to be perhaps $50,000 : in other words, if her job were not dangerous, I might have to pay her a lot of money to have her agree to subject herself to the danger. On the other hand, if she were forced to have a dangerous job, and the only way of avoiding it was bribing me, what would she pay? She's only earning $30,000 per year. Probably having a dangerous job is better than starvation. So she might be willing to bribe me only $15,000. Hence the answer to my second question might be much larger than the answer to my first question. This same phenomenon, due to income effects, will occur, on a much smaller scale, if I ask about the cost of a tax. Typically, people will be willing to pay less to avoid a tax than they will require in compensation in order to accept a tax.

Graphically, if I consider a tax on food, and there is only one other good, then a tax on food

will tilt the budget line of the consumer. So if food consumption is measured on the horizontal axis, then introducing a tax on food makes the budget line steeper, and pivots it about its intercept on the vertical axis. When the person has the usual shape of indifference curve, introducing the tax induces her to choose a different consumption bundle, and moves her to a lower indifference curve. (It moves her from the solid red higher indifference curve, to the dotted green lower indifference curve, in Figure 8.)

What would happen if she bribed me not to introduce the tax? Bribing me causes her income to fall. Her budget line shifts in parallel. Because she has given up some income (by giving it to me), she now must choose another consumption bundle, and she winds up with lower utility. If she bribes me enough, then she winds up so much worse off that she's on the same indifference curve that she would be on if the tax had been introduced. That bribe, the one that moves her exactly to this (low) indifference curve is the answer to my first question. Call the bribe B. Bribing me B, and having no tax imposed, bring her to the same indifference curve I, as having the tax introduced (and not bribing me). B is the largest bribe that she would be willing to pay, to prevent the tax. This amount B is called the "equivalent variation" to introducing the tax.

If the price of clothing were \$1 per unit, then this equivalent variation is about \$8 in Figure 8. Her income was about \$24. (That's the vertical intercept of her original budget line.) Shifting her budget line in parallel, until it is tangent to the lower indifference curve, reduces the income to about \$16. That difference is the bribe that she would just be willing to pay.

Alternatively, suppose I introduce the tax, and then compensate her by some amount C to compensate for the damage done by the tax. This amount C is the answer to my second question, and is called the "compensating variation" to the introduction of the tax. How much is it? Introducing the tax pivots the budget line in around the vertical intercept, moving her from her original high indifference curve I' to a lower indifference curve I. To compensate her, I give her some money, which shifts her budget line **out** parallel. The parallel shift out moves her to a new choice of consumption, on a higher indifference curve than I. If I compensate her enough, then her budget line shifts out so much that the new line leads her to choose a consumption bundle on the same indifference curve I' as she was on before the tax was introduced (and before the compensation was made). Putting in the tax, and then giving her C dollars, leave her exactly as well off as she was originally.

In Figure 8, the compensating variation looks to be about \$12. Her initial income is \$24. Compensating her, after the price increase, shifts her budget line out, from the dashed light–blue line to the dotted purple line. The new shifted–out budget line has a vertical intercept of about 36, meaning that the compensation I had to give her, to get her back up to the higher indifference curve, was $36 - 24 = 12$.

In general, if the price of good Y, the good on the vertical axis, is \$1 per unit, then the amount B equals the distance I shifted her original budget line down, in going from indifference curve I' to indifference curve I. The amount C equals the distance I shifted her new budget line up, to get tangent to the original (high) indifference curve I'. If food, the taxed good, is a *normal* good, then

the amount C is actually larger than the amount B.

The more commonly method of measuring these equivalent and compensating variations uses not the indifference curve diagram, but a totally different diagram, a demand curve diagram. These measures of the cost of a tax can be calculated using the area under a demand curve for the taxed good.

Specifically, the amount B, one of my measures of the cost of the tax, is the area under the person's *compensated* demand curve for food, between the pre–tax price and the post–tax price. So it's exactly the area *abcd* in figure 9 — if the demand curve is a compensated demand curve. You remember what is a compensated demand curve? It shows how the quantity demanded of food varies with the price of food, not when the person's *income* is held constant, but when the person's *utility* is held constant. That is, it measures how her food consumption varies with the price of food if she is compensated so as to stay on the same indifference curve. Which indifference curve? If I am measuring amount B, the equivalent variation to the tax, the amount she would have to bribe me not to introduce the tax, then the indifference curve is I, the (lower) indifference curve she would be on if the tax were introduced. If I am measuring C, the compensating variation to the tax, the amount I would have to compensate her in order to undo the damage of the tax, then the indifference curve is I', the higher indifference curve she was on before any tax was contemplated.

When food is a normal good, that is when a person's food consumption increases with income, then her compensated demand curve shifts out as her utility increases (just as her ordinary demand curve shifts out as her income increases). So the compensated demand curve corresponding to the higher indifference curve I' is higher than the compensated demand curve corresponding to I. That means that the compensating variation C to the tax is higher than the equivalent variation B, since it is the area under a higher demand curve.

And the area under the "ordinary" demand curve? It is somewhere in between B and C. So it's not a bad approximation to the cost of the tax — but it is not the exact answer to either of my questions.

And my "naïve" answer, the amount of the tax times the amount of food she buys? In figure 9, the amount of food she buys if there is a tax is *cd*, and the amount of food she buys if there is no tax is *ab*, so that the area under the demand curve between the two prices lies somewhere between the tax times the "old" amount of food she buys and the tax rate times the "new" amount of food she buys.