The general form of the Ramsey rule, equation (12), was used in the previous section to yield two rules for optimal taxes : the "equi–proportional" rule (17) and the "inverse elasticity" rule (25).

But it can be used as well to derive some implications for optimal commodity taxation when **some goods are not taxed**. Why is that case relevant? In the section on Taxation and Efficiency ("Excess Burden is Almost Everywhere"), it was argued that a uniform sales tax (or an income tax) was inefficient because it was not really taxing all goods which mattered to the consumer : her consumption of leisure is untaxed. Leisure is difficult to measure. And when individuals choose the amount (or intensity) of hours of labour that they supply, then leisure will be left untaxed if all "purchased" commodities are taxed. That's why there is an excess burden to income taxation (or sales taxation).

So suppose that some good, called L (for "leisure") cannot be taxed, and we are looking for the optimal tax rates on the remaining 2 commodities X and Y which can be taxed. Equations (9) and (10) still define the relation among the tax rates on the commodities which can be taxed, even when there are more than 2 goods (some of which are untaxed). I will repeat those equations here :

$$(\lambda - 1)X = -\lambda[t_x \frac{\partial X^H}{\partial P_X} + t_y \frac{\partial Y^H}{\partial P_X}] \tag{9}$$

$$(\lambda - 1)Y = -\lambda[t_x \frac{\partial X^H}{\partial P_Y} + t_y \frac{\partial Y^H}{\partial P_Y}] \tag{10}$$

If I denote the tax **rate** on a good by τ, so that

$$\tau_x = \frac{t_X}{P_X} \quad ; \quad \tau_Y = \frac{t_Y}{P_Y} \tag{26}$$

then (9) and (10) become

$$(\lambda - 1)X = -\lambda[\tau_x \frac{\partial X^H}{\partial P_X} P_X + \tau_y \frac{\partial Y^H}{\partial P_X} P_Y] \tag{27}$$

$$(\lambda - 1)Y = -\lambda[\tau_x \frac{\partial X^H}{\partial P_Y} P_X + \tau_y \frac{\partial Y^H}{\partial P_Y} P_Y] \tag{28}$$

Letting η_{ij} denote the compensated elasticity of demand for good i with respect to the price of good j[1],

$$\eta_{Xi} \equiv \frac{\partial X}{\partial P_i}\frac{P_i}{X} \quad \eta_{Yi} \equiv \frac{\partial Y}{\partial P_i}\frac{P_i}{Y} \qquad (i = X, Y) \tag{29}$$

equations (27) and (28) become

$$\frac{\lambda - 1}{\lambda} = -\tau_X \eta_{XX} - \tau_Y \eta_{XY} = -\tau_X \eta_{YX} - \tau_Y \eta_{YY} \tag{29}$$

[1] so that $|\eta_X|$, defined in the previous section, equals $-\eta_{XX}$

so that

$$\tau_X = \frac{\eta_{XY} - \eta_{YY}}{\eta_{YX} - \eta_{XX}} \tau_Y \tag{31}$$

At this point, another property of compensated demand functions will be needed :

RESULT $\sum_j \frac{\partial X^H}{\partial P_j} P_j = 0$ where the summation takes place over **all** goods (taxed and untaxed).

$$\textit{why } \sum_j \frac{\partial X^H}{\partial P_j} P_j = 0$$

Only relative prices matter. So if all prices were to increase by the same proportion, the compensated (Hicksian) demands for any good would not change, since the slope of the budget line does not change. So if we change each price P_j by $\Delta P_j = aP_j$ for some constant a, compensated demand for good X would not change. So the overall effect of these price changes on compensated demand for good X,

$$\sum_j \frac{\partial X^H}{\partial P_j} \Delta P_j = a[\sum_j \frac{\partial X^H}{\partial P_j} P_j] = 0$$

which proves the result.

The above result can also be written

$$\eta_{XX} + \eta_{XY} + \eta_{XL} = 0 \tag{32}$$

$$\eta_{YX} + \eta_{YY} + \eta_{YL} = 0 \tag{33}$$

where the three goods are X, Y, and untaxed leisure L, or

$$\eta_{XY} = -\eta_{XX} - \eta_{XL} \tag{34}$$

$$\eta_{YX} = -\eta_{YY} - \eta_{YL} \tag{35}$$

Substituting from (34) and 35) into (31)

$$\tau_X = [\frac{-(\eta_{XX} + \eta_{YY}) - \eta_{XL}}{-(\eta_{XX} + \eta_{YY} - \eta_{YL}}] \tau_Y \tag{36}$$

When there is a third untaxed good ("leisure" in the most important example), this new version of the Ramsey rule, equation (36) says that the relative tax rates on the two taxed goods X and Y depend on the elasticities of substitution η_{XL} and η_{YL} between the taxed goods and the untaxed good. Specifically, equation (36) says that if good L cannot be taxed, but if the tax rates on the other goods are set optimally, then good X should be taxed at a higher rate than good Y, if good X is a net complement to L and good Y is a net substitute for L

So the above result, which is due to economists named **Corlett** and **Hague**, suggests that we should not tax all taxable commodities at the same rate when leisure cannot be taxed. Instead, goods associated with leisure (movies, video games, home repair equipment) should be taxed at higher rates than substitutes for leisure (work clothes, professional training). The reason is that the income tax distorts people's leisure–labour choices, by making them substitute leisure for labour, since leisure consumption is untaxed. Taxing complements to leisure (or subsidizing substitutes for leisure) will reduce the extent of this distortion.

Now return to the case of independent demands, in which the inverse elasticity form of the Ramsey rule applies. This rule says that commodity tax rates should be higher for commodities with price–inelastic demands, and lower for commodities with price–elastic demands. That rule would then imply very high tax rates on food, and particularly on basic staples such as bread, potatoes, rice. These are commodities for which the compensated own–price elasticity of demand is usually estimated to be quite small. On the other hand, there are quite a few luxuries for which the compensated own–price elasticity of demand appears to be fairly high.

So the inverse elasticity form of the Ramsey rule would imply a commodity tax system which could be very **regressive**. That is because the only concern in deriving the optimal commodity tax rules was **efficiency**. **Equity** across people was not considered at all.

In fact, the derivation of these optimal commodity tax rules was done for **a single consumer**. That is, the measure of excess burden is the area under an individual consumer's compensated demand curve (or the value of the expenditure function for an individual consumer).

This is a huge shortcoming of the Ramsey rules. Since there are many consumers in the Canadian economy, we have to somehow compare the damage done to different consumers by the tax. Should we consider the loss of \$1 due to a tax by person 1 as exactly the same as the loss of \$1 by person 2? What if the two people have very different income?

If we do care only about efficiency, then the problem of optimal commodity taxation could be regarded as a bit artificial. Recall that there are taxes with no excess burden, taxes that are efficient. Any **lump–sum** tax will have no excess burden. The problem with lump–sum taxes is that the only practical lump–sum taxes are **head taxes** (and perhaps some taxes that are similar to head taxes). A head tax is simply a tax of \$x per person, regardless of the person's income, consumption pattern, age, sex, or education. That is a very easy tax to administer. It is efficient. The problem with it is that it is very very regressive.

So if we really do not care at all about equity, only efficiency, then we could use a head tax to raise government revenue. This has less excess burden than the optimal commodity taxes defined by the Ramsey rules. (Those optimal commodity taxes do have excess burden : it's just that they have the smallest excess burden among all commodity tax systems.) Why might we prefer commodity taxes to head taxes? One reason might be that commodity taxes, although they may be regressive, are not as regressive as head taxes. So if we cared about equity, as well as efficiency, then we might prefer commodity taxes to head taxes.

However, if we do care about equity, then we must modify the rules for optimal commodity taxation. If we care about equity, then it matters who is bearing the burden of taxes. Other things equal, we will prefer commodity taxes on goods which are consumed mostly by higher–income people. So there is a potential conflict here between equity and efficiency.

The Ramsey rules can be modified to take into account this conflict. That is, we can derive optimal commodity tax rules when the harm done by taxes to low–income people is regarded as a bigger problem than an equivalent amount of harm done to higher–income people. The rules are more complicated that the (relatively) simple equi–proportional or inverse–elasticity Ramsey rules. The tax rate on a good now depends on its distributional characteristics (that is, how consumption of the good varies with people's income), as well as on the elasticity of its compensated demand.

So although the basic Ramsey rules derived here provide some useful indications for the design of commodity taxation, they must be modified if income distribution is a concern.

Of course in Canada, and in most developed economies, the taxes which seem to affect income redistribution the most are not commodity taxes, but taxes on personal income. Designing the shape of the income tax schedule is another sort of optimal tax problem, one quite different from Ramsey's optimal commodity taxation problem. This optimal income tax problem is the subject of the next part of this section.