## The Personal Income tax : (f) Taxation and Saving

The basic model used here to analyze the saving decision is the ( familiar, I hope ) twoperiod model, used in intermediate microeconomics ( chapter 10 of Varian, [ part of ] chapter 25 in Nicholson ). In this model, a person is assumed to choose how much to consume in each of two periods, the present period and the future period. The person is also assumed to know not only what she earns in the present period, but what she will earn in the future period. These income levels, $Y_{P}$ in the present period, and $Y_{F}$ in the future period are also assumed to be exogenously given. The person's problem is to choose consumption levels $C_{P}$ in the present period and $C_{F}$ in the future period, given her known income stream $\left(Y_{P}, Y_{F}\right)$. To repeat, the income stream is what she has, the consumption stream is what she is choosing.

The person also knows the rate of return on savings, $r$. If she saves 1 dollar in the present period, she will get $1+r$ dollars in the future period. This rate of return $r$ is also taken as given.

However, on many forms of saving, the net return is subject to personal income taxation in Canada. If a person saves 1 dollar in a bank account (or certificate of deposit, or bond), then the $r$ dollars interest she earns will be subject to tax (when it accrues). That means that what she will get in the future period, if she saves a dollar in an asset on which the return is taxed, is $1+r-r t=1+[1-t] r$ dollars, if $t$ is her marginal income tax rate.

She has an income of $Y_{P}$ in the present period. What she does not consume in the present period, she saves.

$$
\begin{equation*}
S=Y_{P}-C_{P} \tag{1}
\end{equation*}
$$

where $S$ is the total amount of saving she does in the present period. Since this is a model of a world which lasts for only two periods, she will cash in her savings in the future period, and use the money to pay for more consumption in that period. If the net return to saving is taxed, then

$$
\begin{equation*}
C_{F}=Y_{F}+(1+[1-t] r) S \tag{2}
\end{equation*}
$$

(There are two sources of money for future period consumption : income earned during the future period, and income from the previous period's saving.) Substituting for $S$ from equation (1) into equation (2) yields

$$
\begin{equation*}
C_{F}=Y_{F}+(1+[1-t] r)\left(Y_{P}-C_{P}\right) \tag{3}
\end{equation*}
$$

which can be re-written

$$
\begin{equation*}
C_{F}+(1+[1-t] r) C_{P}=Y_{F}+(1+[1-t] r) Y_{P} \tag{4}
\end{equation*}
$$

or

$$
\begin{equation*}
\frac{C_{F}}{1+[1-t] r}+C_{P}=\frac{Y_{F}}{1+[1-t] r}+Y_{P} \tag{5}
\end{equation*}
$$

Equation (5) says that the present value of the person's consumption, discounted using the after-tax rate of return, must equal the present value of the person's income.

Now if the following terms are defined :

$$
\begin{gathered}
P_{P} \equiv 1+[1-t] r \\
P_{F} \equiv 1 \\
M \equiv Y_{F}+(1+[1-t] r) Y_{P}
\end{gathered}
$$

then equation (5) becomes

$$
\begin{equation*}
P_{P} C_{P}+P_{F} C_{F}=M \tag{6}
\end{equation*}
$$

which looks exactly like the equation of a person's budget line. Here $P_{P}$ is the price of present consumption, in terms of future consumption, and the price of future consumption $P_{F}$ is taken as the unit of account. Notice that $M$, the right hand side of equation (6), is exogenous to the person. It also depends on the price $P_{P}$.

Equation (6) is a budget line with slope $P_{P} / P_{F}$, if consumption in the present is graphed on the horizontal axis and consumption in the future is graphed on the vertical, as in figure 2 , (or in figures 18.7 - 18.10 in Rosen et al). (In figure 2, the endowment point is an income profile of 80 in the present, and 20 in the future, and the rate of return is set at 60 percent, reflecting the fact that the periods are a lot longer than 1 year, if we are considering a lifetime saving decision. Taxation of the return to saving pivots the budget line about the endowment point ( 80,20 ).)

The person's endowment point $\left(Y_{P}, Y_{F}\right)$ is on this budget line. That is, the combination $C_{P}=Y_{P}, C_{F}=Y_{F}$ satisfies equation (5). One feasible consumption plan for the person is simply to consume as she earns, neither saving nor borrowing.

From the definition of $P_{P}$ above, an increase in the tax rate on the net return to saving will lower $P_{P}$, the price of current consumption, making the budget line less steep, and pivoting it around the endowment point $\left(Y_{P}, Y_{F}\right)$.

What will the person choose to do if her return to saving is taxed at a higher rate? She wants to pick the consumption plan $\left(C_{P}, C_{F}\right)$ which she prefers most, from all the feasible plans - that is, from all the plans which satisfy the budget constraint. She regards consumption in each period as a "good", giving her downward-sloping indifference curves. The slope of the indifference curves represents the rate at which she is willing to trade off consumption in one period in exchange for consumption in the other period. That means that she choose a point on the budget line at which her indifference curve is tangent to the budget line. If this tangency is above and to the left of the endowment point, then $C_{F}>Y_{F}$ and $C_{P}<Y_{P}$ : the person has positive saving as she is trying
to transfer income from the present period to the future. If the tangency is below and to the right then the person is borrowing.

Now $P_{P} \equiv 1+[1-t] r$ is the price of present period consumption, in terms of future consumption. An increase in the tax rate $t$ therefore represents a fall in the price of present period consumption, relative to future consumption. (If the net return $[1-t] r$ falls, then the amount of future period consumption that you must give up to get a given increase in present period consumption falls.) We might expect that a fall in the price of present period consumption would lead to an increase in the level of present period consumption - just as a fall in the price of coffee would lead to an increase in the level of consumption of coffee, unless coffee were a Giffen good.

But $M$, the right hand side of equation (6) [ and the value of the person's income stream, viewed from the perspective of the future ] falls as well when $t$ increases ( at least when $Y_{F}>0$ ). That means that there is an additional "income effect" : the decrease in $M$ means that people will want to consume less $C_{P}$, if $C_{P}$ is a normal good.

The overall impact of the change in $P_{P}$ on present period consumption can be represented by a modified Slutsky equation

$$
\begin{equation*}
\left.\frac{\partial C_{P}}{\partial P_{P}}=\frac{\partial C_{P}}{\partial P_{P}} \right\rvert\, \text { compensated }+S \frac{\partial C_{P}}{\partial M} \tag{7}
\end{equation*}
$$

where the first term on the right hand side of equation (7) is the compensated own-price derivative of demand for present period consumption, and where the second term is the income derivative of present period consumption demand times the amount of saving ( see also section 10.4 of Varian ).

The first term on the right side of equation (7) must be negative. Compensated own-price derivatives are always negative. But the second term will be positive if $S>0$, and if present period consumption is a normal good. In that case, the overall effect of an increase in $P_{P}$ on $C_{P}$ could go either way : negative if the first effect, the substitution effect, dominated, and positive if the second effect, the income effect, dominated.

Figure 3 depicts a case where the substitution effect dominates, and the person saves less when the return to saving is taxed than when it is not taxed. In the figure, the person chooses a level of present consumption of 50 when her savings is not taxed, so that she is saving $80-50=30$, leading to a future consumption level of 68 . Introduction of a $40 \%$ tax on the return to saving moves the tangency down and to the right in the diagram : the new (lower) indifference curve is tangent to the dotted budget line at $(58.8,48.8)$, meaning that her saving has fallen from 30 to $80-58.8=21.2$ because of the lower return. Figure 4 depicts the opposite case, in which the income effect dominates. In figure 4, the person's optimal consumption combination (the tangency of her indifference curve with the budget line) shifts straight down (more or less), although actually slightly to the left, when the return to saving is taxed, so that she saves (slightly) more when interest income is taxed. (The actual tangencies are $(70.27,35.56)$ when the return is not taxed, and ( $69.60,34.15$ ) when the return is taxed, so that taxation increases her saving by about 6 percent.)
(Figures 18.9 and 18.10 in Rosen et al are similar to figures 3 and 4.)
What does this ambiguity say about the effects of taxation of saving? First of all, an increase in $t$ corresponds to a decrease in $P_{P}$. Secondly, from equation (1), an increase in $C_{P}$ is exactly the same as a decrease in $S$ ( more spending today means less saving ). So if the substitution effect dominates, taxation of saving leads to less saving. But if $S>0$, and if the income effect dominates, then taxation of the return to saving might actually lead the person to save more (as in figure 4).

This latter case can be motivated by thinking about target savers. These are people for whom the substitution effect is small, so that the income effect will dominate. This sort of person seeks to save enough so as to achieve a certain target level of future consumption $C_{F}$. (Notice that, in figure 4 , the person's second-period consumption changes very little when the net return to saving changes a lot, so that she is close to being a target saver.) If taxation lowers her net return to saving, then she will have to save more in the present period in order to reach her target in the future period. Such a person, therefore, would save more when saving is taxed than when it is not.

Assuming that present period consumption is normal, then the ambiguity would disappear if the person were a borrower, in which case $S<0$. Then both terms on the right side of equation (7) are negative : a decrease in the net cost of borrowing must lead the person to borrow more.

However, the tax treatment of borrowing introduces a further complication. The return to certain forms of saving is taxed. But, in general the cost of borrowing is not tax deductible. ( It would be tax-deductible if the proceeds were invested in income-earning assets, but not if the proceeds were used for current consumption, or the purchase of RRSPs. ) So suppose that the bank lets a person borrow at the same rate $r$ as the rate she would earn on her savings. Even so, her after-tax net rate of return on saving is $[1-t] r$ since the return to saving is taxable, but her after-tax cost of borrowing is still $r$, since she cannot deduct borrowing costs from her income. What this lack of deductibility of borrowing costs means is that the price of current consumption $P_{P}$ equals $1+[1-t] r$ to the left of the endowment point $\left(Y_{P}, Y_{F}\right)$ so that she is saving, but that the price $P_{P}$ equals $1+r$ to the right of the endowment point, where she is borrowing. Figure 5 (as well as figure 18.11 in Rosen et al) depicts the kinked budget line which arises in this case.

Now regardless of the sign ( or magnitude ) of the effect of taxation of interest income on saving, there will be an excess burden from this taxation. The excess burden depends on the compensated demand derivative. Even if the income and substitution effects exactly cancelled, so that taxation did not affect saving, there would be an excess burden due to the taxation of interest income. For this reason, some economists have proposed replacing the personal income tax with a progressive expenditure tax, which is basically an income tax with all interest and capital gains income untaxed. An expenditure tax would get rid of the excess burden from the taxation of interest income. However, that certainly does not mean that there would be no excess burden if interest income were not taxed. In the model presented here, I simplified by assuming that the person's income was given. In reality, people's income depends on their choices, and will be affected by an income tax - or by an expenditure tax. There still will be an excess burden due
to the taxation of labour income, even if the return to saving is not taxed. And if the return to saving is not taxed, the tax rates will have to be increased, to make up for the lost revenue. This increase will make larger the excess burden from taxation of labour income. So it is not a foregone conclusion that replacing the income tax with an expenditure tax would reduce the overall excess burden of the tax system : it gets rid of one excess burden "triangle", but makes another excess burden "triangle" larger.

Finally, a note about actual Canadian practice in the taxation of the return to saving. Only some forms of saving are taxed. Buying a house ( or paying down a mortgage on an existing house ) is a form of saving, and the return to this form of saving is not taxed. Buying stocks is a form of saving, and part of the return to this form of saving ( the capital gains one might earn ) is taxed at a much lower effective rate than labour income. People can also save using Registered Retirement Savings Plans (RRSPs). Contributions to RRSPs are deductible from present tax. The total return ( interest plus principal ) is taxable if one cashes in an RRSP before reaching age 65. Nonetheless, even so this tax treatment makes the effective rate of return of saving in an RRSP equal to $1+r$ : if I put $\$ 1$ in an RRSP this year, and cash in the RRSP next year, then current consumption only goes down by $1-t$, since I get a tax deduction of $\$ 1$ this year ; when I cash in the RRSP next year, I pay tax on the whole $1+r$, so my future consumption goes up by $(1+r)(1-t)$. Therefore, if I put another dollar in an RRSP in the present period, and cash it in in the future period, then $\Delta C_{F} / \Delta C_{P}=-(1+r)(1-t) /(1-t)=-(1+r)$ : the slope of my budget line will be the same as if there were no tax.

In other words, there is an enormous advantage to using an RRSP for investment, even if one has no intention whatsoever of saving the return until after retirement. If what I put in the plan is tax deductible, and what I take out is fully taxed - interest and principal - then the calculation above says that my effective return, net of all taxes is $r$. So the return to saving in an RRSP is exempt from taxation, even if the taxpayer cashes in the whole RRSP long before retirement.

There are further advantages - not considered further here - to people whose income fluctuates, so that their marginal tax bracket varies from year to year. All the calculations in this section were based on a single, constant marginal tax rate $t$. Quite a bit of profitable tax arbitrage can be achieved by such strategies as contributing to an RRSP in years when one faces a high marginal rate, and cashing in the RRSP in a year in which one faces a low marginal rate.

