AS/ECON 4070 Answers to Mid–term Exam October 2003

1. Fixed coefficients in production means that the isoquants are L–shaped : no substitution is possible between inputs. That is, the amount of labour, and of capital, used to produce a given quantity of output, do not vary with the relative prices of labour and capital.

So there is no factor substitution effect from a tax on the use of labour in the food industry.

To determine the incidence of this tax, only the output effect need be considered. If the food industry is less labour intensive than other industries, then the output effect of a tax which is levied only in the food industry will reduce the relative return to other inputs, and increase the relative return to labour.

With no substitution effect, and an output effect which harms the other input(s) to production, then labour will bear less than its share of the tax. The incidence will fall disproportionately on other inputs.

2. Given the demand function, the person's quantity demanded of clothing would be $3600/[20^2] = 9$ if there were no tax on clothing, and $3600/[30^2] = 4$ if there were a fifty percent tax on clothing (driving the consumer price P_Y up to 30).

The tax revenue collected would be 40, since the tax is 10 per unit, and 4 units are consumed (when clothing is taxed).

The exact measure of the excess burden is the compensating variation to the tax, minus the tax revenue. The compensating variation is the area inside the compensated demand curve, between the no-tax price of 20 and the tax–inclusive price of 30, so

$$CV = \int_{20}^{30} 3600(\frac{P_X}{P_Y})^2 dP_Y$$

Since the integral of $1/P_Y^2$ is $-1/P_Y$, therefore

$$CV = 3600/20 - 3600/30 = 180 - 120 = 60$$

so that the excess burden is $60 - 40 = 20$.

A reasonably good approximation can be obtained by pretending that the compensated demand curve is a straight line, so that the excess burden is the area of a triangle, between the prices of 20 and 30, and the quantities of 4 and 9, an area of $(1/2)(30-20)(9-4) = 25$. This is not a bad approximation, but it overstates the area since the actual demand curve lies below the line connecting $(4, 30)$ with $(9, 20)$.

Another approximation is $(1/2)\eta p_Y Y(\tau_Y)^2$ (equation 16.3 of the text), where η is the compensated price elasticity of demand, and τ_Y is the proportional tax rate. Evaluating the elasticity

$$\eta = -\frac{\partial Y}{\partial P_Y}\frac{P_Y}{Y} = -(2)3600\frac{P_X^2}{P_Y^3}(P_Y)Y = 2$$

Since $\tau_Y = 0.5$, this formula gives the excess burden as $(1/2)(2)(20)4(1/2)^2 = 20$, which is actually exactly right in this case.

3. The statement is **false**. A proportional sales tax on all goods and services is the same as a proportional wage tax (for a person who earns all of her income by working). So adding a wage tax to an already–existing sales tax is just like increasing the proportional sales tax further, and will increase the excess burden.

When people choose how much to work, a proportional sales tax is equivalent to a subsidy on leisure : the amount of consumption foregone by working a little less will decline because of the tax. A wage tax is also a subsidy on leisure : it reduces the rate of take–home pay from w to $w(1-t)$ so it reduces the price of leisure.