Q1. Suppose a tax on hotel accommodation were introduced, of \$10 per night per room. How much would the (tax included) price of hotel accommodation increase, if the hotel industry were competitive, and had a supply curve with the equation

$$q^s = 5000 + p_s$$

and a demand curve with the equation

$$Q^D = 6000 - 9P^D$$

where p_s is the price per room per night received by hotel owners, P^D is the price per room per night paid by customers, q^s is the quantity of rooms supplied per night, and Q^D is the quantity of rooms demanded per night?

A1. There are several ways to do this.

One is to solve for the equilibrium. In equilibrium, quantity demanded equals quantity supplied. If the tax is t per hotel room, then $P^D = p_s + t$, so that the equations of the supply and demand curve imply that

$$5000 + (P^D - t) = 6000 - 9P^D$$

or

$$P^D = 100 + \frac{t}{10} \qquad (1-1)$$

That means that, if there were no tax, the tax–included price P^D of accommodation would be \$100 per night ; if a tax of \$10 per night were introduced, then the tax–included price would increase to $100 + 10/10 = 101$.

Alternatively, you can just plug in the formula for linear supply and demand curves. If the demand curve has the equation $A - BP^D$, and the supply curve the equation $-c + dp_s$, then the tax–included price is

$$\frac{A+c}{B+d} + \frac{d}{B+d}t \qquad (1-2)$$

In this question $A = 6000$, $B = 9$, $c = -5000$ and $d = 1$. Plugging into equation $(1-2)$ gives equation $(1-1)$.

A third method is to look at the relative slopes of the supply and demand curves. Here

$$\frac{\partial Q^D}{\partial P^D} = -9$$

$$\frac{\partial q_s}{\partial p_s} = 1$$

so that demanders bear a share

$$\frac{\partial q^s/\partial p_s}{\partial q/\partial p_s - \partial Q^D/\partial P^D} = \frac{1}{10}$$

of the tax. That says that, if a tax of \$10 is introduced, the tax–included price goes up by \$1.

Finally, the elasticity formula gives a good approximation. The share of the tax born by buyers is approximately

$$\frac{\epsilon_s}{\epsilon_D + \epsilon_s}$$

Evaluating elasticities at the original, no–tax equilibrium ($p = 100$, $Q = 5100$),

$$\epsilon_s = \frac{\partial q_s}{\partial p_s}\frac{p_s}{q_s} = (1)\frac{100}{5100} = \frac{1}{51}$$

$$\epsilon_D = -\frac{\partial Q^D}{\partial P^D}\frac{P^D}{Q^D} = -(-9)\frac{100}{5100} = \frac{9}{51}$$

implying (again) that demanders bear 10 percent of the tax.

Q2. Discuss how workers and capital owners would bear the cost of an excise tax on good X, in the Harberger (2 sector, 2 input) model of general equilibrium tax incidence.

A2. The question referred to an **excise** tax on a commodity. The incidence of an excise tax in the Harberger model depends on which good is more labour intensive – in other words on the "output effect" used in the analysis of a specific factor tax.

So the short answer is : the tax will raise the real wage, and lower the real return to capital, if the taxed sector X is more capital–intensive than the untaxed sector. If the taxed sector is more labour–intensive, then the tax will raise the real return to capital and lower the real wage.

Why does the effect depend on the relative factor intensities? Putting an excise tax on good X shifts in the demand curve for the good (as buyers have to pay the tax, on top of the price received by sellers). So, if input prices did not change, production of X would decrease, and production of Y would increase (since the relative price of the untaxed good Y will have fallen).

Since inputs are assumed perfectly mobile between industries, an increase in Y production, and a decrease in in X production, are accomplished by some labour and capital moving from the X industry to the Y industry. If the X industry is more capital intensive than the Y industry, then the demand for capital would fall, and demand for labour rise, from this shift in production : production is increasing of the good which uses lots of labour per unit, and production is decreasing of the good which uses lots of capital per unit. These changes in demand would then cause the input prices to adjust. If X were more capital intensive, the return to capital would have to fall, to eliminate the excess supply of unemployed capital, and the wage would fall, to eliminate the excess demand for labour.

Since the prices of inputs are not being taxed, there is no substitution effect here. The incidence of the tax depends on the relative factor intensities across industries.

Q3. What would be the excess burden of a unit tax of \$1 on good Y, if the net–of–tax price were \$3 (and its supply was perfectly elastic), and if the price of good X were \$1 per unit, for a person (with total income 100) whose preferences could be represented by the utility function

$$U(X,Y) = X + 24\sqrt{Y}$$

where X and Y are the quantities she consumes of the two goods?

A3. To get the answer here, the person's demand function for the taxed good must be obtained. To do so, use the result that any consumer chooses a consumption bundle (X,Y) such that $MU_X/MU_Y = P_X/P_Y$, where MU_X and MU_Y are the marginal utilities, and P_X and P_Y are the (tax–included) prices paid by the person. Here

$$MU_X = 1$$
$$MU_Y = \frac{12}{\sqrt{Y}}$$

so that (when $P_X = 1$), the consumer's choice of consumption bundle must obey the equation

$$\frac{1}{12/\sqrt{Y}} = \frac{1}{P_Y}$$

or

$$\sqrt{Y} = \frac{12}{P_Y}$$

implying that

$$Y = \frac{144}{(P_Y)^2} \qquad (3-1)$$

equation $(3-1)$ is the consumer's demand function for good Y, since it expresses the quantity demanded as a function of the price faced by the consumer. Here the quantity demanded does not depend on the person's income, since her preferences are quasi–linear.

If there were no tax on good Y, then P_Y would be 3, and the person would buy (from equation $(3-1)$) $144/9 = 16$ units of good Y. She would spend $P_Y Y = 3(16) = 48$ on good Y, leaving her $100 - 48 = 52$ for good X. So her utility would be $X + 24\sqrt{Y} = 52 + 24\sqrt{16} = 148$.

A \$1 tax on good Y increases its price to \$4, so that the person demands $144/16 = 9$ units, costing her $(9)(4) = 36$. That leaves her $100 - 36 = 64$ to spend on good X, for a utility of $64 + 24\sqrt{9} = 136$.

The tax has lowered her utility by 12. What income decrease would be equivalent in its effect? Since her demand for Y does not depend on income, and changes in income will translate directly into changes in X consumption. If her income falls by m dollars, so will her X consumption.

Therefore, the equivalent variation to the tax on Y consumption is 12 : a tax of \$1 on good Y, or losing \$12, both have the same effect on her utility, a fall from 148 to 136. (Here, because of the quasi–linearity, 12 is also the compensating variation to the tax.)

The tax collects \$9 in revenue : she consumes 9 units of Y when it is taxed, and the tax is \$1 per unit.

Therefore, the excess burden of the tax is $12 - 9 = 3$.