AS/ECON 4070 3.0AF Answers to Midterm Exam October 2005

*Q*1.“The fewer firms that there are in an industry, the more an excise tax will be shifted forward onto consumers”. True, false or uncertain? Explain briefly.

*A*1. The short answer here is “uncertain” : an industry with few firms may actually shift **less** of the tax forward onto consumers than an industry with many firms (and the same technology and demand curve), but it also may shift more.

Some examples :

(*i*) If an industry has a straight–line demand curve, and a constant marginal cost, then 100% of the tax will be shifted forward onto consumers if the industry is perfectly competitive, while only 50% will be shifted if the industry is a (single–price) monopoly. So going from many firms (perfect competition) to very few firms (1 firm ; monopoly) reduces the proportion of the tax which is shifted forward.

(*ii*) If the industry were an oligopoly, and had the same shape demand and cost curves as in example (*i*), then the proportion of the tax which is shifted forward will decrease with the number of firms in the oligopoly — if the oligopoly is a Cournot oligopoly, in which firms choose quantities simultaneously and non–cooperatively.

(*iii*) If an industry has a **constant elasticity** demand curve, with the elasticity greater than 1, and (again) constant marginal costs, then 100% of an excise tax will be shifted forward onto consumers if the industry is perfectly competitive, but now **more** than 100% will be shifted forward if the industry is a single–price monopoly. Here more of the excise tax is shifted forward in the industry with fewer firms.

(*iv*) If firms in an oligopoly collude, so as to maximize their combined profits, then they behave just like a monopoly : the incidence of a tax would not vary with the number of colluding firms.

(*v*) In a Bertrand oligopoly, in which firms choose prices simultaneously and non–cooperatively, the equilibrium price (if the average cost is constant) is the same as in a perfectly competitive industry, no matter how few firms there are in the oligopoly. Here again the incidence of the tax would not vary with the number of firms in an industry.

*Q*2. Discuss as precisely as possible the incidence of a tax on the use of labour in the manufacturing industry if the manufacturing industry is relatively capital intensive, and at the same time it is very easy to substitute labour for capital in other industries (other than manufacturing).

*A*2. The Harberger model of general equilibrium tax incidence is probably the best model to analyze this question. In that model, labour (or any other input) is perfectly mobile between industries. Thus the incidence of a tax will fall on labour in general or capital in general, or both.

In the Harberger model, a specific factor tax, such as a tax on the use of labour in the manufacturing industry, has 2 effects, an **output effect** and a **substitution effect**. The output effect is a decline in the relative price of the input used more intensively in the taxed industry. Here the taxed sector (manufacturing) is more capital intensive, so that the output effect implies an increase in labour's wage, relative to the return to capital. The substitution effect is a decline in the relative price of the factor being taxed. Here labour is taxed, so that the substitution effect works in the opposite direction to the output effect : a decline in the wage relative to the return to capital.

Overall, the two effects work in opposite directions. But if the elasticity of substitution is relatively high in other industries, then the output effect will be small. Why? A small decline in the relative price of capital will induce other industries to substitute lots of capital for labour, enabling the other industries to absorb easily all the capital moving from the manufacturing industry.

Therefore, in this example, the incidence of a tax on the use of labour in the manufacturing industry is likely to fall mostly on labour (in general).

*Q*3. If a person's preferences could be represented by the utility function

$$u(X,Y) = X + 24\sqrt{Y}$$

(where X and Y are her consumption of 2 goods), what would be the excess burden of a \$1 tax on the person's Y consumption, if initially the prices of goods X and Y were \$1 each, and if the person's income were 200?

*A*3. The first step is to find the consumer's demand function for good Y. To do so, use the fact that she chooses a consumption bundle such that her marginal rate of substitution between Y and X equals the relative prices :

$$MRS_{YX} = \frac{P_Y}{P_X}$$

Here

$$MU_X = \frac{\partial u}{\partial X} = 1$$

$$MU_Y = \frac{\partial u}{\partial Y} = \frac{12}{\sqrt{Y}}$$

so that the consumer chooses a bundle for which

$$MRS_{YX} = \frac{MU_Y}{MU_X} = \frac{12}{\sqrt{Y}} = \frac{P_Y}{P_X}$$

or

$$Y = 144(\frac{P_X}{P_Y})^2$$

Here the price of good X is 1, so that the consumer's demand for good Y is

$$Y = \frac{144}{(P_Y)^2}$$

The tax of \$1 increases the consumer's price of good Y from 1 to 2, and therefore reduces her demand from 144 to 36.

The tax also collects revenue of \$36 : \$1 per unit, and the consumer chooses to buy 36 units when the tax is \$1.

There are several ways to calculate the excess burden. One exact measure is to use the area inside the demand curve, minus the tax revenue. So

$$EB = \int_1^2 \frac{144}{(P_Y)^2} dP_Y - 36$$

Since the integral of the function $1/(P_Y)^2$ is $1/P_Y$, therefore

$$EB = \int_1^2 \frac{144}{(P_Y)^2} dP_Y - 36 = \frac{144}{P_Y}|_2^1 - 36 = \frac{144}{1} - \frac{144}{2} - 36 = 36$$

36 is exactly the excess burden of the tax here.

An approximation to the answer is to assume that the demand curve is a straight line (which it actually is not), and use the formula for the area of a triangle. Here the triangle has a width of $144 - 36 = 108$, the change in quantity demanded, and the height is $2 - 1 = 1$, the tax, so that the formula gives an answer of $(0.5)(1)(108) = 54$, higher than the actual value since the demand curve is "bowed in" compared to a straight line.

Another formula is

$$EB \approx \frac{1}{2}\tau^2\eta^C PQ$$

where τ is the tax rate, as a proportion of the final price, and η^C is the elasticity of demand. Here the elasticity of demand is 2 (since $Q_Y = 144/(P_Y)^2$). The tax rate τ equals 1/2, P equals 2, and Q equals 36, so that this approximation formula yields an excess burden of $\frac{1}{2}(\frac{1}{2})^2(2)(2)(36) = 18$, an underestimate of the true answer.

A variation on this formula is that $EB \approx (1/2)\eta^C\tau$ times the tax revenue. Here the tax revenue is 36, so that this variation gives an excess burden of $(1/2)(2)(1/2)(36) = 18$ again.

Formula 16.3 in the text uses the tax rate as a percentage of the net–of–tax price, which here is $1/1 = 1$, the net–of–tax price 1, and the *before*–tax quantity, 144, so that

$$EB \approx \frac{1}{2}(2)(1)(144)(1)^2 = 72$$

which is a pretty rough approximation.

Finally, we could calculate directly the compensating variation. When there was no tax, the person consumed 144 units of Y, and $200 - 1(144) = 56$ units of X, for a utility of

$$U = 56 + 24\sqrt{144} = 344$$

With a tax, she chooses $Y = 36$, so that $X = 200 - 2(36) = 128$, and her utility is

$$U = 128 + 24\sqrt{36} = 272$$

How much money would we have to give her to get her utility back up to 344? If we gave her 72 dollars, then she could spend it all on good X (keeping her Y consumption at 36), giving her utility of $128 + 72 + 24\sqrt{36} = 344$. So the compensating variation to the tax increase is 72 dollars, and the excess burden is the CV minus the tax revenue, or $72 - 36 = 36$.