AS/ECON 4070 3.0AF Answers to Midterm Exam October 2006

$Q1$. What would be the incidence of a \$7 per unit excise tax on a product which is provided in a perfectly competitive market, for which the demand curve has the equation

$$Q^D = 120 - 5P^D$$

and for which the supply curve had the equation

$$Q_s = 2p_s - 20$$

where P^D is the price paid by demanders, p_s is the price received by suppliers, Q^D is the quantity demanded and Q_s is the quantity supplied?

$A1$. If t is the unit tax on the good, then $P^D = p_s + t$. In equilibrium, the quantity demanded must equal the quantity supplied, so that

$$120 - 5(p_s + t) = 2p_s - 20$$

or

$$7p_s = 140 - 5t$$

implying that

$$p_s = 20 - \frac{5}{7}t$$

When there is no tax, $t = 0$, so that $p_s = P^D = 20$. With a \$7 tax, $t = 7$, so that $p_s = 15$ and $P^D = 15 + 7 = 22$. Therefore, demanders bear a fraction 2/7 of the tax, and suppliers bear a fraction 5/7.

Since both the demand curve and the supply curve are linear here, this answer could also be obtained by plugging in the formula

$$p_s = A + c - \frac{B}{B+d}t$$

or

$$P^D = A + c + \frac{d}{B+d}t$$

when the demand curve has the equation $Q^D = A - BP^D$ and the supply curve has the equation $Q_s = dp_s - c$. Here $A = 120$, $B = 5$, $c = 20$ and $d = 2$.

Alternatively, the elasticities of supply and demand give a good approximation of the incidence. If there were no tax, so that $p_s = P^D = 20$ and $Q_s = Q^D - 20$, then

$$\epsilon^D D = -\frac{\partial Q^D}{\partial P^D}\frac{P^D}{Q^D} = 5\frac{10}{20} = 2.5$$

$$\epsilon_s = \frac{\partial Q_s}{\partial p_s}\frac{p_s}{Q_s} = 2\frac{10}{20} = 1$$

so that the elasticity formula predicts demanders will bear a share

$$\frac{\epsilon_s}{\epsilon_s + \epsilon^D} = \frac{1}{3.5}$$

of the tax — which happens in this case to be exactly correct.

*Q*2. What would be the incidence of a tax levied on all revenue earned by firms in the service sector in Canada? State as precisely as possible the assumptions that you are making.

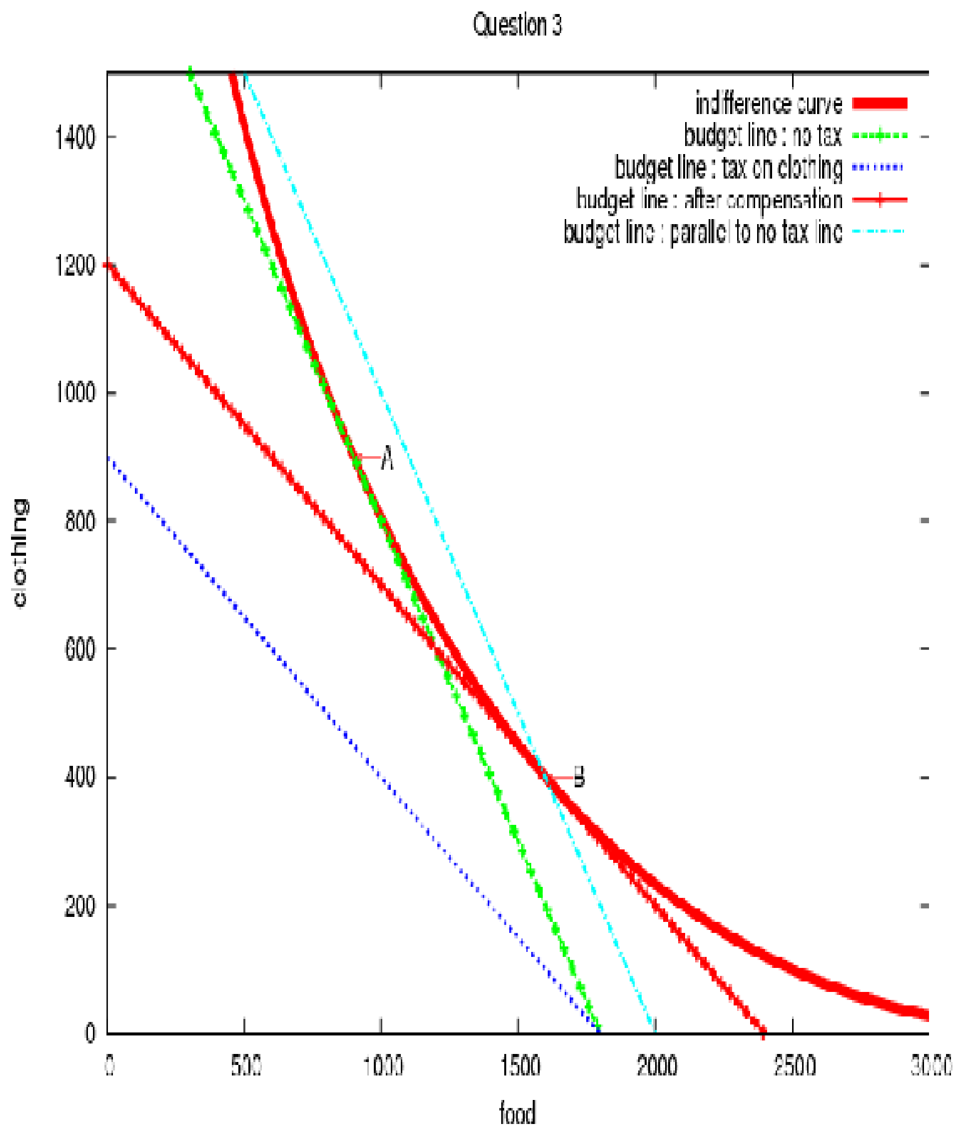
*A*2. The Harberger model is one general equilibrium model which can be used to derive the incidence of an excise tax on a large sector of the economy. The key assumptions of the Harberger model are : fixed total endowments of each factor of production in Canada ; perfect mobility of all factors between the service sector and other sectors ; constant returns to scale in production in every sector ; perfect competition in all factor and goods markets ; identical consumption patterns among all factor owners in Canada.

Given those assumptions, the tax can be born only by factor owners in general (not by owners of a factor which is specific to some sector).

Given these assumptions, the incidence of any excise tax will be a fall in a relative return to the factor used most intensively in the taxed sector. It seems reasonable to assume that the service sector is relatively labour–intensive compared to other industries such as manufacturing or mining, so that the Harberger model predicts that the incidence of the excise tax will fall on workers in Canada, through a fall in their net wage.

Other assumptions would give rise to other conclusions. If we do not assume perfect factor mobility among sectors, then a tax on the output of the service sector might be born predominantly by owners of factors specific to that sector : perhaps workers whose skills are suitable only for service jobs.

An excise tax can also be analyzed using supply–demand analysis : an example of partial equilibrium analysis. In this case, the tax would be shifted forward onto consumers of services, if the elasticity of supply of services were much greater than the price–elasticity of demand for services. But partial equilibrium analysis is probably less appropriate here than general equilibrium analysis : a large fraction of Canadians' consumption expenditure is spent on services ; as well, many Canadians earn their incomes from working in the service sector. Partial equilibrium analysis does not take account of the effects an excise tax has in other markets, nor of changes in markets for factors of production.


Question 3
indifference curve
budget line : no tax
budget line : tax on clothing
budget line : after compensation
budget line : parallel to no tax line
clothing
food
A
B
0
200
400
600
800
1000
1200
1400
0
500
1000
1500
2000
2500
3000

$Q3$. The picture on the previous page shows the effects of a \$1 tax on clothing, when the price of food is \$1, and when the net–of–tax price of clothing is \$1. Point A, at $(900, 900)$ is the consumption bundle which the person would choose if there were no tax. Point B at $(1600, 400)$ is on the same indifference curve as A, but is located where the slope of the indifference curve is $-1/2$.

State, as precisely as possible : (a) the person's compensated elasticity of demand for clothing ; (b) the compensating variation to the tax ; (c) the revenue the tax would collect if the person were compensated for the damage done by the tax; (d) the excess burden of the tax.

$A3$. (a) The person's demand for clothing fell from 900 to 400 when the (tax–included) price of clothing rose from \$1 to \$2. The best approximation we have for the compensated own–price elasticity of demand for clothing is

$$\eta^C = \frac{\Delta C}{\Delta P_C}\frac{P_C}{C}$$

Here $\Delta C = 900 - 400 = 500$ and $\Delta P_C = 2 - 1 = 1$. If we use the "before tax" values of $P_C = 1$ and $C = 900$, then

$$\eta^C = \frac{500}{1}\frac{1}{900} = 0.5556$$

If we use the "after tax" values of $P_C = 2$ and $Q_C = 400$, then

$$\eta^C = \frac{500}{1}\frac{2}{400} = 2.5$$

A better approximation is to take the average of the before– and after–tax prices, and the average of the before– and after–tax quantities : $P^C = \frac{1+2}{2}$ and $C = \frac{400+900}{2}$ so that

$$\eta^C = \frac{500}{1}\frac{1.5}{650} = 1.154$$

Why is this third answer a better approximation? The quantity consumed of clothing fell to just under half what it had been, when the price to the consumer doubled. So the compensated elasticity of demand appears to be a little greater than 1. (Or : note that the consumer's expenditure on clothing goes down slightly, from 900 to 800, when its price increases. That implies a price elasticity a little greater than 1.)

(b) The person's income originally, when there was no tax, and when $P_C = P_F = 1$, was $900 + 900 = 1800$. How much income does the consumer need to afford the bundle B, when $P_F = 1$ and $P_C = 2$? The cost of the bundle (including tax) is $(1)(1600) + (2)(400) = 2400$, so she needs an income of 2400. Therefore, she needs to be compensated with an income increase of 600 to keep her on the same indifference curve, when P_C rises from 1 to 2. $CV = 600$. [The horizontal intercept of the line through A with slope -1, and the line through B with slope $-1/2$, is exactly the CV, and is 600 in the picture.]

(c) The person consumes 400 units of clothing at the point B, which is the consumption she chooses after the tax is impose (and after she has been compensated with \$600). Since the tax is \$1 per unit of clothing, \$400 is the tax revenue collected. [The distance between the horizontal intercept of the line with slope -1 through B, and that of the line with slope $-1/2$ through B also measures this revenue, and it is 400 in the picture.]

(d) The excess burden is the difference between the compensating variation to the tax increase, and the tax revenue : $EB = 600 - 400 = 200$.