AS/ECON 4070 3.0AF Answers to Midterm Exam October 2007

$Q1$. How would the burden of a 100% tax (expressed as a fraction of the "before–tax" price) be split between buyers and sellers, in a perfectly competitive market in which the quantity demanded is

$$Q^D = 75 - 2(P^D)^2$$

and the quantity supplied is

$$Q^s = (p_s)^2$$

where P^D is the price paid by buyers and p_s is the price received by sellers?

$A1$. There are (at least) 2 ways to solve this question.

One way is to solve explicitly for the prices, with and without the tax.

Let τ represent the tax rate (as a proportion of the seller's price p_s). Since, given this definition,

$$P^D = p_s(1+\tau)$$

in equilibrium, the condition that quantity supplied equal quantity demanded can be written

$$75 - 2[p_s(1+\tau)]^2 = (p_s)^2 \tag{1}$$

or

$$(p_s)^2[1 + 2(1+\tau)^2] = 75 \tag{2}$$

When there is no tax, $\tau = 0$, so that (2) becomes

$$3(p_s)^2 = 75$$

or

$$p_s = P^D = 5$$

With a 100% tax, $\tau = 1$, so that (2) becomes

$$(p_s)^2[1 + 2(2)^2] = 75$$

or

$$(p_s)^2 = \frac{75}{9}$$

which means that $p_s \approx 2.886$ and $P^D = (1+\tau)p_s \approx 5.573$ when there is a 100 percent tax. Therefore, explicit calculation shows that the tax lowers the price received by sellers by $5 - 2.886 = 2.114$, and increases the price paid by buyers by about $5.573 - 5 = 0.573$. So buyers bear about 20 to 25 percent of the tax (since $0.573/(0.573 + 2.114) = 0.213$).

Another way to estimate the incidence is to use an approximation formula, based on the elasticities of supply and demand. Given the equations for the supply and demand curves,

$$\epsilon_s \equiv \frac{\partial Q_s}{\partial p_s}\frac{p_s}{Q_s} = 2 \tag{3}$$

$$\epsilon_D \equiv -\frac{\partial Q^D}{\partial P^D}\frac{P_D}{Q_D} = (4P^D)\frac{P^D}{Q^D} = 4\frac{(P^D)^2}{Q^D} \tag{4}$$

But since $P^D = p_s(1+\tau)$, and since $Q^D = Q_s = (p_s)^2$ in equilibrium, equation (4) can be written

$$\epsilon_D = 4(1+\tau)^2 \tag{5}$$

Now what share of a tax do the buyers bear? The elasticity formula says that buyers's share of the tax burden is approximately

$$S_D = \frac{\epsilon^s}{\epsilon_s + \frac{1}{1+\tau}\epsilon_D} \tag{6}$$

Plugging in from (3) and (5), the buyers' share of the tax burden is

$$S_D = \frac{2}{2 + 4(1+\tau)} = \frac{1}{1 + 2(1+\tau)} \tag{7}$$

The tax goes from 0 to 100%. Evaluating the elasticity formula (7) at $\tau = 0$ and at $\tau = 1$ gives a value for this share of the burden S_D of $\frac{1}{3}$ and $\frac{1}{5}$ respectively, which bracket the actual value of about 21 %.

Similar results can be obtained using an approximation formula involving the **slopes** of the demand and supply equations :since the absolute value of the slope of the demand equation goes from twice the slope of the supply equation to 4 times the slope of the supply equation, this formula as well predicts that buyers will bear a much smaller share of the burden than sellers.

$Q2$. Describe, as precisely as possible, the incidence of a proportional tax on all wages and salaries paid in service industries.

$A2$. The most useful model to analyze the sort of *specific factor tax* described in the question is the Harberger general equilibrium tax incidence model.

If we assume that there are 2 sectors in the economy, the service sector and "everything else", and that there are two inputs to production, labour and capital, then a tax on wages and salaries paid in the service sector is a partial factor tax on the use of L in sector X, where L denotes "labour", and X is the service sector.

Harberger's assumptions (in particular, perfect mobility of factors between industries, fixed total factor supplies, and identical consumption patterns for everyone) imply that the burden of a tax must fall on the return to labour (everywhere in the economy) and on the return to capital (everywhere in the economy).

How much of this burden is born by labour depends on two effects. The "substitution effect" results from firms in the service sector substituting other inputs for labour, due to the tax. This effect must lower the relative return to labour. The "output effect" results from the contraction of the service sector, due to the imposition of the tax. This effect must lower the relative return of the factor which is used most intensively in the taxed sector.

So, if the service sector is relatively more labour–intensive than the economy as a whole, then the output effect and the substitution effect work in the same direction : labour's share in national income must fall because of the tax. This case seems more likely, since services tends to require a lot of human input.

However, if the service sector actually were **less** labour–intensive than the rest of the economy, then the output effect would work to increase labour's share in national

income, going in the opposite direction to the substitution effect. Whether labour or capital bore a larger share of the burden of the tax would — if the service sector were less labour–intensive than the rest of the economy — depend on how big the output effect was, compared to the substitution effect.

$Q3$. Is the local property tax a regressive tax, or a progressive tax? Explain your answer.

$A3$. Whether the property tax is regressive or progressive depends mostly on how much of the tax is shifted forward, onto consumers of housing, or backwards onto capital owners.

If most of the burden of the property tax is shifted forward onto consumers of housing, then the tax will appear quite regressive. The share of housing expenditure in income is much higher, on average, for low–income people than for high–income people. So if the burden of the tax is proportional to people's housing expenditure, then the burden – as a share of people's income — will fall with income, making the tax look regressive.

This regressivity will be reduced if lifetime expenditure and income data are used, instead of annual data.

On the other hand, a general equilibrium approach (similar to that of the Harberger model) is used, then much of the burden of the property tax will be shifted backwards, onto capital owners. (These are capital owners in general, not just investors in real estate, since capital is very mobile among different industries.) This certainly will be the case if the construction industry is relatively capital–intensive, and if the aggregate supply of capital in the economy is fixed.

Capital income, as a share of income, is highest among the richest income groups. (That is, not only do rich people, on average, have higher earnings from capital than poorer people, these earnings are higher as a fraction of their overall income.) So a tax born by owners of capital will be progressive.

The extent to which the tax can be shifted backwards onto capital owners may be limited if the supply of capital to the country as a whole is not fixed, but elastic.

Land is an important input into construction as well (which is why a more complicated model than Harberger's 2–factor model may be needed to analyze general equilibrium

incidence of the property tax). The ownnership of land, as well, is concentrated among the upper income brackets, so that any shifting backwards of the burden of the property tax onto land owners will make the tax appear quite progressive in its incidence.