$Q1$. What would be the incidence of a \$2 unit tax in a perfectly competitive market in which the demand curve had the equation

$$Q^d = \frac{30}{P^D}$$

and the supply curve had the equation

$$Q^s = 4 - \frac{10}{p_s}$$

where Q^D is the total quantity demanded of the good, Q^s is the total quantity supplied of the good, P^D is the price paid by buyers and p_s is the price received by sellers?

$A1$. Equilibrium in a perfectly competitive market requires quantity supplied to equal quantity demanded, which means here that

$$\frac{30}{p_s + t} = 4 - \frac{10}{p_s} \tag{1}$$

where t is the unit tax on the good, and I have used the fact that $P_D = p_s + t$. If both sides are multiplied by $p_s(p_s + t)$, equation (1) can be written

$$4(p_s)^2 - (40 - 4t)p_s - 10t = 0 \tag{2}$$

When there is no tax, equation (2) becomes

$$4(p_s)^2 = 40p_s \tag{3}$$

so that the equilibrium price is $p_s = P^D = 10$ when there is no tax at all. With a tax of \$2, equation (2) becomes

$$4(p_s)^2 - 32p_s - 20 = 0 \tag{4}$$

which has a solution of

$$p_s = 4 + \frac{1}{2}\sqrt{84} \approx 8.5826 \tag{5}$$

which means that sellers bear a fraction $(10 - 8.5826)/2 = 0.7087$ of the tax. So the nearly exact answer is that sellers bear 70.87 % of the tax, and buyers bear the other 29.13 % $(= (8.5826 + 2 - 10)/2)$.

But this incidence can be approximated, using the elasticity formulae. When there is no tax, $p_s = P^D = 10$ and $Q^d = Q^s = 3$. Therefore, the own–price elasticity of demand is

$$\eta^d = -\left(\frac{\partial Q^d}{\partial P^D}\right)\frac{P^D}{Q^D} = \frac{30}{10^2}\frac{10}{3} = 1 \tag{6}$$

and the price elasticity of supply is

$$\eta^s = \frac{\partial Q^s}{\partial p^s}\frac{p_s}{Q^s} = \frac{10}{(10)^2}\frac{10}{3} = \frac{1}{3} \tag{7}$$

The fraction of the tax born by sellers should be

$$\frac{\eta^d(p_s/P^D)}{\eta^d(p_s/P^D) + \eta^s} \tag{8}$$

which equals $1/(1 + 1/3) = 3/4$ when $t = 0$ and $p_s = P^D = 10$. So the elasticity approximation formula says that sellers should bear 75 percent of the tax, which is close to the exact answer.

A similar elasticity formula uses the derivatives of the demand and supply functions, instead of the elasticities. Since

$$\frac{\partial Q^D}{\partial P^D} = \frac{30}{(P^D)^2}$$

and

$$\frac{\partial Q^s}{\partial p^s} = \frac{10}{(p_s)^2}$$

sellers should bear a fraction

$$\frac{-\frac{\partial Q^D}{\partial P^D}}{-\frac{\partial Q^D}{\partial P^D} + \frac{\partial Q^s}{\partial p^s}}$$

of the tax burden. This fraction also equals $3/4$ when $t = 0$ and $p_s = P^D$.

*Q*2. What would be the incidence of replacing the Canadian federal goods and service tax (GST) with a payroll tax [i.e. a proportional tax on people's labour income] which raised the same amount of revenue? Explain your answer.

*A*2. The goods and service tax is a proportional tax on nearly all of people's consumption. (There are a few exempt categories, such as food purchased in a supermarket.)

So the policy change is to replace a tax on (nearly) all consumption, with a tax on labour income.

A proportional tax on everyone's consumption is equivalent to a proportional tax on all people's income.

Therefore, the change is (nearly) the same as replacing a tax on all income, with an equal–yield tax on labour income.

If input supplies in the Canadian economy were completely fixed, then a tax on the income earned from some factor would fall completely on the owners of that factor. So — if factor supplies were fixed — a tax on labour income would be born by workers, in proportion to the income they earn from their jobs. Under this assumption, a tax on all income (or on all consumption) would be split among workers, capital owners, and owners of other inputs, in proportion to their income from these inputs.

Therefore, if all factor supplies were fixed, then the proposed change would be born by labour owners. Owners of other factors would gain, since their share of the cost of the GST would be shifted to workers.

On the other hand, if factor supplies to Canada are not fixed, then the incidence of a general income tax — or of a general consumption tax — would not necessarily be shared by owners of all factors. In particular, if capital is perfectly mobile internationally, a tax on capital income is born by owners of less mobile factors : land and labour. Therefore, if capital is perfectly mobile, labour would bear nearly all the cost of a tax on capital income. A shift from a proportional consumption tax to a proportional labour income tax would not have much effect, since nearly all the incidence of each of the taxes would be on labour owners.

*Q*3. Is the local property tax a regressive tax, or a progressive tax? Explain your answer.

*A*3 Whether the property tax is regressive or progressive depends mostly on how much of the tax is shifted forward, onto consumers of housing, or backwards onto capital owners.

If most of the burden of the property tax is shifted forward onto consumers of housing, then the tax will appear quite regressive. The share of housing expenditure in income is much higher, on average, for low–income people than for high–income people. So if the burden of the tax is proportional to people's housing expenditure, then the burden – as a share of people's income — will fall with income, making the tax look regressive.

This regressivity will be reduced if lifetime expenditure and income data are used, instead of annual data.

On the other hand, a general equilibrium approach (similar to that of the Harberger model) is used, then much of the burden of the property tax will be shifted backwards, onto capital owners. (These are capital owners in general, not just investors in real estate, since capital is very mobile among different industries.) This certainly will be the case if the construction industry is relatively capital–intensive, and if the aggregate supply of capital in the economy is fixed.

Capital income, as a share of income, is highest among the richest income groups. (That is, not only do rich people, on average, have higher earnings from capital than poorer people, these earnings are higher as a fraction of their overall income.) So a tax born by owners of capital will be progressive.

The extent to which the tax can be shifted backwards onto capital owners may be limited if the supply of capital to the country as a whole is not fixed, but elastic.

Land is an important input into construction as well (which is why a more complicated model than Harberger's 2–factor model may be needed to analyze general equilibrium incidence of the property tax). The ownership of land, as well, is concentrated among the upper income brackets, so that any shifting backwards of the burden of the property tax onto land owners will make the tax appear quite progressive in its incidence.