Q1. What would be the incidence of a $\$ 5$ unit tax in a perfectly competitive market in which the demand curve had the equation

$$
Q^{d}=32-P^{D}
$$

and the supply curve had the equation

$$
Q^{s}=4 p_{s}-8
$$

where $Q^{D}$ is the total quantity demanded of the good, $Q^{s}$ is the total quantity supplied of the good, $P^{D}$ is the price paid by buyers and $p_{s}$ is the price received by sellers?
$A 1$. If $t$ is the unit tax on the good, then $P^{D}=p_{s}+t$. In equilibrium, the quantity demanded must equal the quantity supplied, so that

$$
32-\left(p_{s}+t\right)=4 p_{s}-8
$$

or

$$
5 p_{s}=40-t
$$

implying that

$$
p_{s}=8-\frac{1}{5} t
$$

When there is no tax, $t=0$, so that $p_{s}=P^{D}=8$. With a $\$ 5 \operatorname{tax}, t=5$, so that $p_{s}$ decreases to 7 and $P^{D}$ increases to $7+5=12$. Therefore, demanders bear $80 \%$ of the tax, and suppliers bear $20 \%$.

Since both the demand curve and the supply curve are linear here, this answer could also be obtained by plugging in the formula

$$
p_{s}=A+c-\frac{B}{B+d} t
$$

or

$$
P^{D}=A+c+\frac{d}{B+d} t
$$

when the demand curve has the equation $Q^{D}=A-B P^{D}$ and the supply curve has the equation $Q_{s}=d p_{s}-c$. Here $A=32, B=1, c=8$ and $d=4$.

Alternatively, the elasticities of supply and demand give a good approximation of the incidence. If there were no tax, so that $p_{s}=P^{D}=8$ and $Q_{s}=Q^{D}-24$, then

$$
\begin{aligned}
\epsilon^{D} & =-\frac{\partial Q^{D}}{\partial P^{D}} \frac{P^{D}}{Q^{D}}=\frac{8}{24}=\frac{1}{3} \\
\epsilon_{s} & =\frac{\partial Q_{s}}{\partial p_{s}} \frac{p_{s}}{Q_{s}}=4 \frac{8}{24}=\frac{4}{3}
\end{aligned}
$$

so that the elasticity formula predicts demanders will bear a share

$$
\frac{\epsilon_{s}}{\epsilon_{s}+\epsilon^{D}}=\frac{4}{5}
$$

of the tax - which happens in this case to be exactly correct.

Q2. Discuss the long-run incidence of a tax on land used for housing. That is, the tax would be levied on owners of houses, based on the total land area of the property ; land used in other sectors (commercial, agricultural, office buildings, etc.) would not be subject to the tax.
$A 2$. The tax in question would be an example of what is often called a "specific factor tax", a tax on the use of a factor of production, but only in some specific sector.

The Harberger model is a useful model to analyze the incidence of this sort of tax. In the long run, factors of production are mobile among sectors, as owners of land try to find the use which maximizes their income. Even in the long run, the aggregate supply of land is quite inelastic, so that Harberger's assumption that all factor supplies are fixed is not a bad one.

Two effects can be distinguished which determine the backwards incidence of a specific factor tax, the substitution effect and the output effect. The substitution effect is a fall in the return to the factor which is being taxed. Here that is land. The output effect is a fall in the return to the factor used most intensively in the sector in which the tax is levied. Here that sector is the housing industry. Now the housing industry is quite land-intensive. The share of land costs in total housing costs can be well above $50 \%$ in metropolitan areas such as Toronto. And certainly land costs account for a much smaller
share than $50 \%$ for many other industries (services, manufacturing, transportation). So it seems reasonable to assume that housing is relatively land-intensive. That being the case, the output and substitution effects work in the same direction : the net return to land should fall, relative to the return to other factors of production.
[If housing were less land-intensive than the rest of the economy, then the return to land might even rise relative to the return to other factors. But that does not seem to be a very reasonable assumption.]

Therefore, the Harberger model suggests that land owners should bear the cost of a tax on the use of land in residential housing. Note that the mobility of factors among sectors means that it is land owners in general, including the owners of land used for commercial or industrial purposes, who bear the tax. Not just owners of residential land.

The Harberger model ignored forward shifting of taxes onto consumers of housing. Here the inelasticity of land supply suggests that the tax will indeed be shifted backwards, rather than forward onto consumers of housing.

Q3. Suppose that an excise tax is imposed on some good $Y$, and consumers regard some other untaxed good $X$ as a very close substitute (but not a perfect substitute) for good $Y$. Would there be an excess burden from the tax on $Y$ ? Explain briefly.
$A 3$ The short answer here is there will be an excess burden, and that the excess burden will be large relative to the revenue collected by the tax.

Suppose that there were only 2 goods, $X$ and $Y$. Figures 1 and 2 in the "Taxation and Efficiency" section of the lecture notes, indicate that the magnitude of the excess burden depends on how far to the right the point $B$ is, compared with point $A$ : that is, how much of a change there will be in the person's consumption bundle when the tax is imposed, and when she is compensated so as to stay on the same indifference curve. If $X$ and $Y$ were perfect complements then $A$ and $B$ would coincide : there would be no excess burden. If $X$ and $Y$ were "nearly" complementary, so that the slope of the indifference curve changed a lot as we moved a small way along it, then a big tax change (and the associated compensation) would cause just a small change in the
person's consumption bundle, so that the excess burden would be small relative to the tax revenue collected.

But when the two goods are good substitutes, the indifference curve's slope does not change much as we move along it. We would have to move a long way to go from point $A$, where the slope is $-p_{X} / p_{Y}$, to point $B$, where the slope is $-p_{X} /\left(p_{Y}+t_{Y}\right)$. And that means that the excess burden will be big, since the excess burden in figure 1 is proportional to the horizontal distance between $A$ and $B$.

Alternatively, the compensated demand curve representation of excess burden, in figure 4, for example, shows that making the compensated demand curve more elastic, which means making it less steep, will increase the excess burden, relative to the tax revenue collected, for a given tax.

Finally, a less formal answer comes from the fact that the excess burden is caused by the distortion in the consumer's behaviour caused by the tax. When demand is elastic, a given tax will cause a bigger change in the consumer's behaviour : a bigger fall in her $Y$ consumption and a bigger corresponding increase in her $X$ consumption. So more elastic demand leads to more distortion in behaviour which means a bugger excess burden.

Finally, consider what happens to tax revenue if there is a very good substitute for the taxed good $Y$. Taxpayers reduce their consumption of good $Y$ drastically, switching to untaxed $X$ instead. So the tax revenue collected is going to be very small : people aren't going to buy much of the good $(Y)$ which is subject to the tax. The excess burden is the difference between the cost of the tax to the consumer, and the tax revenue collected. Since the tax revenue collected is small, then the excess burden will be large.

