**AP/ECON 4070 3.0AF    Answers to Midterm Exam    October 2013**

*Q*1. Is it possible that a \$1 excise tax on some good could lead to the total price paid by buyers of the good increasing by **more** than \$1? Explain briefly.

*A*1. If the good is produced in a perfectly competitive industry, then the answer is "no". In a perfectly competitive industry, the change in the demand price $P_D$ in response to a tax increase can be written

$$\frac{\partial P_D}{\partial t} = \frac{S'(p_s)}{S'(p_s) - D'(P_D)} \tag{1}$$

where $S'(p_s)$ is the derivative of the quantity supplied with respect to the suppliers' price, and $D'(P_D)$ is the derivative of the quantity demanded with respect to the price paid by demanders. As long as the demand curve slopes down and the supply curve slopes up, the expression in equation (1) must be between 0 and 1 in value, so that a \$1 tax cannot increase the price paid by demanders by more than \$1.

On the other hand, if the market is not perfectly competitive, then it is possible that a tax may be shifted by more than 100 percent. For example, suppose that the good is provided by a monopoly, which charges the same price to all buyers. Suppose further that the marginal cost of production for the good is constant, and equals $c$. Then the price $P$ which maximizes the monopoly's profit, when it has a production cost $c$ and faces a unit tax of $t$ per unit sold, is

$$P = \frac{\epsilon}{\epsilon - 1}(c + t) \tag{2}$$

where $\epsilon$ is the absolute value of the own–price elasticity of demand for the good. If that elasticity is constant, and it exceeds 1[1], then equation (2) shows that a \$1 tax will raise the monopoly's profit–maximizing price by $\frac{\epsilon}{\epsilon-1} > 1$ dollars.

[1] And the elasticity must exceed 1 at the monopoly's profit–maximizing choice of output.

*Q*2. What would be the effect on income distribution in Ontario of a tax change in which workers' compensation premia (which are a proportional tax on wage income) were abolished, and the provincial sales tax rate was increased so as to keep provincial tax revenue the same? Explain your answer.

*A*2. In the simple world of the Harberger model, the assumption that total factor supplies to the whole economy are fixed means that a general factor tax is born entirely by the factor on which it is levied. A general wage tax would be born entirely by workers.

If the provincial sales tax had no exemptions (such as for food bought in grocery stores), then it would correspond to a general sales tax. In the simple world of the Harberger model, a general sales tax is exactly equivalent to a general income tax. A general income tax is born by all sources of income (not just labour), in equal proportion to their income.

Therefore, in the Harberger model, replacing a wage tax (such as workers' compensation premia) by a general sales tax would replace a tax born by labour with a tax born by all factors of production. Thus the change would benefit workers (who bore all of the burden of the wage tax, but only part of the burden of the PST) and harm owners of other factors (who bore none of the burden of the wage tax, and some of the burden of the PST).

In reality, the PST is not a completely general sales tax. It does have some "excise tax" effects : since food is (mostly) exempt, an increase in the PST would cause some expansion of the food industry and contraction of other industries, changes which would benefit workers if (and only if) food production were more labour–intensive than the rest of the economy.

As well, in reality people's consumption patterns are not all identical, as Harberger assumed. The fact that food is mostly exempt from the PST means that an increase in the PST will benefit lower–income people somewhat, since they spend a higher proportion of their incomes on food than do higher–income people.

There are a few more complications. Since people in Ontario shop outside of Ontario (and outsiders do some shopping in Ontario), a general sales tax is not exactly equivalent to a general income tax. A very small fraction of the incidence of the sales tax may be shifted onto these outsiders who buy in Ontario. And Ontarians who shop abroad may bear less of the incidence of a sales tax increase than those who do all of their purchasing in Ontario.

The workers' compensation premia are actually levied only on the first $83,200 of annual income, which makes the tax regressive — the average rate per dollar of income declines with income, for people earning more than $83,200. And the premium rates also vary across industries.

But, broadly speaking, the tax change described in the question would not have a big impact on incidence. Some of the tax burden will be shifted from workers to owners of other inputs (land, capital) — if the supply of these inputs is not very elastic. And the exemptions in the provincial income tax imply some of the burden will be shifted from (lower–income) consumers of PST–exempt goods such as food onto people who spend a lower fraction of their income on these exempt goods.

$Q3$. What is the excess burden (also known as the deadweight loss) of a \$1 tax on clothing, if the price of food were \$1, the original (no–tax) price of clothing were \$1, the consumer's income were \$1000, the consumer's preferences could be represented by the utility function

$$U(F, C) = F + 40\sqrt{C}$$

where $C$ is the quantity consumed of clothing and $F$ the quantity consumed of food, and her compensated demand curve for clothing had the equation

$$C = \frac{400}{(P_C)^2}$$

where $P_C$ is the (tax–included) price that she pays for clothing?

[Note : you will not need to use all the information given above in order to answer the question.]

$A3$. Three possible ways to answer the question : the first two give the exact answer, and the third gives an approximation.

$(i)$. How much would the government have to compensate the consumer for the damage done by the tax?

With no tax, the consumer's demand for clothing is

$$C_{no} = \frac{400}{1} = 400 \tag{3}$$

so that she has 600 dollars to spend on food, and her utility is

$$U_{no} = 600 + 40\sqrt{400} = 1400 \tag{4}$$

With the tax in place, the consumer's demand for clothing is

$$C_{tax} = \frac{400}{2^2} = 100 \tag{5}$$

so that if she has $M$ dollars to spend, she'll spend $(2)(100) = 200$ dollars on clothing, and have $M - 200$ to spend on food, so that her utility will be

$$U_{tax} = M - 200 + 40\sqrt{100} = M + 200 \tag{6}$$

For her utility to be unchanged after the tax and the compensation, she needs a total income $M$ so that $U_{tax} = U_{no}$, or $M = 1200$. Since her original income was 1000 dollars, the required compensation is 200 dollars.

The tax revenue collected is 100 dollars : the consumer chooses to consume $C_{tax} = 100$ units of clothing when there is a tax, and the tax per unit of clothing is \$1.

So the excess burden is the difference between the tax revenue and the required compensation, or 100 dollars.

[Note : It does not matter here whether the equivalent or compensating variation is used to calculate the damage done by the tax. Since the consumer's demand for clothing does not depend on her income, the compensating and equivalent variations are the same.]

$(ii)$. The required compensation is the area under the person's compensated demand curve for clothing, between the "no–tax" price of $P_C = 1$ and the "tax–included" price of $P_C = 2$. The area under a curve graphing a function is the integral of that function. So

$$CV = \int_1^2 D(P_C)dP_C = \int_1^2 \frac{400}{(P_c)^2}dP_C \tag{7}$$

Now the integral is the inverse function of the derivative. So the integral of the function $\frac{400}{(P_C)^2}$ is the function

$$f(P_C) = \frac{400}{P_C} \tag{8}$$

since the derivative of the function $f(P_C)$ is

$$f'(P_C) = \frac{400}{(P_C)^2} \tag{9}$$

Therefore, equation (7) implies that

$$CV = f(1) - f(2) = \frac{400}{1} - \frac{400}{2} = 200 \tag{10}$$

As in part $(i)$, the compensation required for the damage of the tax equals 200 dollars. The tax revenue collected was calculated in part $(i)$ above : it's 100 dollars.

So (again), the excess burden is $200 - 100 = 100$ dollars.

(*iii*). Unlike the previous two methods, which gave an exact measure of the excess burden, this third method gives an approximate answer.

The consumer's demand for clothing was 400 when there was no tax, and was 100 when there was a tax.

The level of the tax is 1 dollar per unit of clothing.

So the excess burden is approximately the area of a triangle, with a height equal to the level of the tax, and the width equal to the change in demand for the taxed good. Here the height is 1, and the width is $400 - 100 = 300$, so the area of the triangle is

$$A = \frac{1}{2}(1)(300) = 150 \tag{11}$$

Notice that this answer — an excess burden of 150 dollars — is not a very accurate approximation, since the demand curve is not a straight line here. The curve $C = \frac{400}{(P_C)^2}$ is "bowed–in" compared to a straight line, so that the area under the demand curve (as calculated in part (*ii*)) is considerably less than the approximation in part (*iii*), which ignores the fact that the curve is bowed in.