AP/ECON 4070 3.0AF   Answers to Midterm Exam   October 2014

$Q1$. What would be the incidence of a \$4 unit tax in a perfectly competitive market in which the demand curve had the equation

$$Q^d = 56 - 3P^D$$

and the supply curve had the equation

$$Q^s = p_s - 8$$

where Q^D is the total quantity demanded of the good, Q^s is the total quantity supplied of the good, P^D is the price paid by buyers and p_s is the price received by sellers?

$A1$. There are several different ways to get the same (correct) answer, which is : suppliers bear 75% of the incidence of the tax (\$3 per unit sold) and demanders bear 25%.

One method is simply to solve for the price of the good, before and after the tax. Since $P^D = p_s + t$, in equilibrium in this market, when quantity demanded equals quantity supplied,

$$Q^d = 56 - 3P^D = 56 - 3(p_s + t) = Q^s = p_s - 8 \qquad (1-1)$$

or

$$56 - 3(p_s + t) = p_s - 8 \qquad (1-2)$$

so that

$$p_s = \frac{64}{4} - 3\frac{t}{4} = 16 - 3\frac{t}{4} \qquad (1-3)$$

When there is no tax, equation $(1-3)$ says that $p_s = P^D = 16$, and when there is a tax imposed of $t = 4$, equation $(1-3)$ says that $p^s = 13$ and $P^D = p_s + t = 17$. So the tax raises the price demanders pay by \$1, and lowers the price suppliers receive by \$3 : suppliers bear $\frac{3}{4}$ of the tax.

Another way of getting the same result is to look at the slopes of the demand and supply functions. The fraction of the tax born by demanders is

$$\frac{\partial P^D}{\partial t} = \frac{\frac{\partial Q^s}{\partial p_s}}{\frac{\partial Q^s}{\partial p_s} - \frac{\partial Q^D}{\partial P^D}} \qquad (1-4)$$

Here $\frac{\partial Q^s}{\partial p_s} = 1$ and $\frac{\partial Q^D}{\partial P^D} = -3$ so that formula $(1-4)$ again says that buyers bear 1/4 of the tax. [Here the approximation formula $(1-4)$ gives an exact answer, because the supply and demand curves are both straight lines.]

A third way of getting this result is to use elasticities : an approximate formula, for the share of the tax born by buyers when the tax is very small, is

$$\frac{\partial P^D}{\partial t} \approx \frac{\epsilon_s}{\epsilon_s + \epsilon_D} \tag{1-5}$$

where ϵ_s and ϵ_D are the absolute values of the own–price elasticities of supply and demand respectively.

In this case

$$\epsilon_s \equiv \frac{\partial Q^s}{\partial p_s}\frac{p_s}{Q^s} = \frac{p_s}{Q^s} \tag{1-6}$$

$$\epsilon_D \equiv -\frac{\partial Q^D}{\partial P^D} = 3\frac{P^D}{Q^D} \tag{1-7}$$

so that the fact that $Q^s = Q^D$ in equilibrium makes formula $(1-5)$ into

$$\frac{\partial P^D}{\partial t} \approx \frac{p_s}{p_s + 3P^D} \tag{1-8}$$

Starting from a situation of no tax at all, so that $p_s = P^D$, formula $(1-8)$ again implies that demanders bear 1/4 of the cost of the tax, and suppliers bear the other 3/4.

*Q*2. What would be the incidence of introducing a new payroll tax — a proportional tax on people's labour income — to fund health care in Ontario, and, at the same time, of reducing the Ontario sales tax rate so as to keep total provincial government revenues constant?

*A*2. Since the question discusses a general sales tax (on all goods), and a labour income tax (on all workers in the province), this is a question about general equilibrium tax incidence.

One of the main results in the section on general equilibrium tax incidence is the equivalence of a general sales tax with a general income tax. In a simple static model of a closed economy, a general sales tax is exactly equivalent to a proportional income tax on **all** sources of income.

So the policy change described in the question is equivalent to the replacement of a proportional tax on **all** income with a proportional tax on **labour** income alone.

For example, if aggregate labour supply in Ontario were fixed, and the aggregate supply of other inputs (such as capital and land) were also fixed, then a proportional income tax would be born by all sources of income, in proportion to the size of the income. And, under these assumptions, a labour income tax would be born (only) by workers, in proportion to the level of their labour incomes.

So one general equilibrium model of the incidence of the tax change described in the question would say that the tax change would shift the some of the burden of taxation from owners of land and capital onto workers.

If supplies of inputs to production were not fixed, the incidence would be different. Workers might be able to shift some of the burden of a labour income tax. Or owners of capital might be able to shift their share of the burden of a general tax on all income.

But — in a static, closed–economy model — it must be true that the original sales tax would have the same incidence as a proportional tax on all sources of income.

*Q*3. What would be the cost to the consumer of a tax of $1 per unit purchased of good Y, and how much revenue would the tax collect, in the following situation?

Initially, the price of good X is 1, and the price of good Y [initially, without the tax] is 1. There is no tax on good X.

The consumer's expenditure function is

$$e(P_X, P_Y, u) = P_X u - 16\frac{(P_X)^2}{P_Y}$$

her compensated ("Hicksian") demand functions for the two goods are

$$X^H(P_X, P_Y, u) = u - 32\frac{P_X}{P_Y}$$

$$Y^H(P_X, P_Y, u) = 16\frac{(P_X)^2}{(P_Y)^2}$$

and her initial level of utility (in the absence of any taxes) is $u = 36$.

*A*3. By definition, the cost to the consumer of the tax is the change in the expenditure required to get to the given level of utility. That is, the cost of the tax on good Y is defined to be

$$e(p_x, p_y + t, u) - e(p_x, p_y, u)$$

where t is the tax on good Y, and p_y the net–of–tax price of good Y.

Here $p_x = p_y = 1$, $t = 1$ and $u = 36$, so that the cost of the tax is

$$e(1, 2, 36) - e(1, 1, 36) \tag{3 – 1}$$

Given the definition of the expenditure function in the question

$$e(1, 2, 36) = 36 - 16\frac{1}{2} = 28 \tag{3 – 2}$$

$$e(1, 1, 36) = 36 - 16\frac{1}{1} = 20 \tag{3 – 3}$$

so that the cost of the tax, to the consumer, is $28 - 20 = 8$.

The tax revenue is the tax per unit of Y, times the number of units of Y that the consumer chooses to buy. From the statement of the question, her quantity demanded is

$$Y^H(1, 2, 36) = 16\frac{1^2}{2^2} = 4 \tag{3 – 4}$$

so that the tax revenue tY equals 4.