AP/ECON 4070 3.0AF   Answers to Midterm Exam   October 2015

$Q1$. What would be the incidence of a \$12 unit tax in a perfectly competitive market in which the demand curve had the equation

$$Q^D = 24 - P^D$$

and the supply curve had the equation

$$Q^s = 2p_s - 12$$

where Q^D is the total quantity demanded of the good, Q^s is the total quantity supplied of the good, P^D is the price paid by buyers and p_s is the price received by sellers?

$A1$. There are several different ways to get the same (correct) answer, which is : suppliers bear 1/3 of the incidence of the tax (\$4 per unit sold) and demanders bear 2/3.

One method is simply to solve for the price of the good, before and after the tax. Since $P^D = p_s + t$, in equilibrium in this market, when quantity demanded equals quantity supplied,

$$Q^d = 24 - P^D = 24 - (p_s + t) = Q^s = 2p_s - 12 \qquad (1-1)$$

or

$$24 - (p_s + t) = 2p_s - 12 \qquad (1-2)$$

so that

$$p_s = \frac{36}{3} - \frac{t}{3} = 12 - \frac{t}{3} \qquad (1-3)$$

When there is no tax, equation $(1-3)$ says that $p_s = P^D = 12$, and when there is a tax imposed of $t = 12$, equation $(1-3)$ says that $p^s = 8$ and $P^D = p_s + t = 20$. So the tax raises the price demanders pay by \$8, and lowers the price suppliers receive by \$4 : suppliers bear $\frac{1}{3}$ of the tax.

Another way of getting the same result is to look at the slopes of the demand and supply functions. The fraction of the tax born by demanders is

$$\frac{\partial P^D}{\partial t} = \frac{\frac{\partial Q^s}{\partial p_s}}{\frac{\partial Q^s}{\partial p_s} - \frac{\partial Q^D}{\partial P^D}} \qquad (1-4)$$

Here $\frac{\partial Q^s}{\partial p_s} = 2$ and $\frac{\partial Q^D}{\partial P^D} = -1$ so that formula $(1-4)$ again says that buyers bear 2/3 of the tax. [Here the approximation formula $(1-4)$ gives an exact answer, because the supply and demand curves are both straight lines.]

A third way of getting this result is to use elasticities : an approximate formula, for the share of the tax born by buyers when the tax is very small, is

$$\frac{\partial P^D}{\partial t} \approx \frac{\epsilon_s}{\epsilon_s + \epsilon_D} \tag{1-5}$$

where ϵ_s and ϵ_D are the absolute values of the own–price elasticities of supply and demand respectively.

In this case

$$\epsilon_s \equiv \frac{\partial Q^s}{\partial p_s}\frac{p_s}{Q^s} = 2\frac{p_s}{Q^s} \tag{1-6}$$

$$\epsilon_D \equiv -\frac{\partial Q^D}{\partial P^D} = \frac{P^D}{Q^D} \tag{1-7}$$

so that the fact that $Q^s = Q^D$ in equilibrium makes formula $(1-5)$ into

$$\frac{\partial P^D}{\partial t} \approx \frac{2p_s}{2p_s + P^D} \tag{1-8}$$

Starting from a situation of no tax at all, so that $p_s = P^D$, formula $(1-8)$ again implies that demanders bear 2/3 of the cost of the tax, and suppliers bear the other 1/3.

*Q*2. Is the local property tax a regressive tax, or a progressive tax? Explain your answer.

*A*2 Whether the property tax is regressive or progressive depends mostly on how much of the tax is shifted forward, onto consumers of housing, or backwards onto capital owners.

If most of the burden of the property tax is shifted forward onto consumers of housing, then the tax will appear quite regressive. The share of housing expenditure in income is much higher, on average, for low–income people than for high–income people. So if the burden of the tax is proportional to people's housing expenditure, then the burden – as a share of people's income — will fall with income, making the tax look regressive.

This regressivity will be reduced if lifetime expenditure and income data are used, instead of annual data.

On the other hand, a general equilibrium approach (similar to that of the Harberger model) is used, then much of the burden of the property tax will be shifted backwards, onto capital owners. (These are capital owners in general, not just investors in real estate, since capital is very mobile among different industries.) This certainly will be the case if the construction industry is relatively capital–intensive, and if the aggregate supply of capital in the economy is fixed.

Capital income, as a share of income, is highest among the richest income groups. (That is, not only do rich people, on average, have higher earnings from capital than poorer people, these earnings are higher as a fraction of their overall income.) So a tax born by owners of capital will be progressive.

The extent to which the tax can be shifted backwards onto capital owners may be limited if the supply of capital to the country as a whole is not fixed, but elastic.

Land is an important input into construction as well (which is why a more complicated model than Harberger's 2–factor model may be needed to analyze general equilibrium incidence of the property tax). The ownership of land, as well, is concentrated among the upper income brackets, so that any shifting backwards of the burden of the property tax onto land owners will make the tax appear quite progressive in its incidence.

*Q*3. What is the excess burden of a 125% tax on clothing, in the following situation? [A tax of 125% means the price of clothing to the consumer is increased from its original price, to 225% of its original price.] The consumer has an expenditure function

$$E(P_F, P_C, u) = \sqrt{P_F P_C}u$$

where P_F is the price paid by the consumer for food, and P_C is the price paid by the consumer for clothing, and u is the consumer's utility (which means that the consumer's "Hicksian", or compensated demand functions for food and clothing are

$$F^H(P_F, P_C, u) = \frac{1}{2}\sqrt{\frac{P_C}{P_F}}u$$

$$C^H(P_F, P_C, u) = \frac{1}{2}\sqrt{\frac{P_F}{P_C}}u \qquad)$$

The initial prices of food and clothing (in the absence of any taxes) are $p_F = 4$ and $p_c = 4$, and the consumer's utility was $u^0 = 18$ if there were no tax, and $u^1 = 12$ if there were a tax (of 125%) on clothing for which the consumer was not compensated.

$A3$. By definition, the cost to the consumer of the tax is the change in the expenditure required to get to the given level of utility. That is, the cost of the tax on good Y is defined to be

$$e(p_x, p_y + t, u) - e(p_x, p_y, u)$$

where t is the tax on good Y, and p_y the net–of–tax price of good Y.

Here $p_x = p_y = 4$, $t = 5$ and $u = 18$ if there were no tax, so that, using the compensating variation, the cost of the tax is

$$e(4, 9, 18) - e(4, 4, 18) \qquad (3-1)$$

Given the definition of the expenditure function in the question

$$e(4, 9, 18) = (\sqrt{4 \cdot 9})(18) = (\sqrt{36})(18) = 108 \qquad (3-2)$$

$$e(4, 4, 18) = (\sqrt{4 \cdot 4})(18) = 72 \qquad (3-3)$$

so that the cost of the tax, to the consumer, is $108 - 72 = 36$.

The tax revenue is the tax per unit of Y, times the number of units of Y that the consumer chooses to buy. From the statement of the question, her quantity demanded is

$$C^H(4, 9, 18) = \frac{1}{2}(\sqrt{\frac{4}{9}})(18) = \frac{1}{3}(18) = 6 \qquad (3-4)$$

so that the tax revenue tY equals $5 \cdot 6 = 30$.

Using the compensating variation, the excess burden of the tax is the difference between the cost of the tax and the tax revenue, or $36 - 30 = 6$.

It is just as good to use the equivalent variation to the tax, which means using the lower reference level of utility 12 which the consumer would get if clothing were taxed and no compensation were made. In this case

$$e(4, 9, 18) = (\sqrt{4 \cdot 9})(12) = (\sqrt{36})(12) = 72 \qquad (3-5)$$

$$e(4, 4, 18) = (\sqrt{4 \cdot 4})(12) = 48 \qquad (3-6)$$

so that the cost of the tax, to the consumer, is $72 - 48 = 24$.

Using the lower utility level of 12 (which would result if the consumer were not compensated) the quantity demanded of the taxed good (clothing) is

$$C^H(4, 9, 12) = \frac{1}{2}(\sqrt{\frac{4}{9}})(12) = \frac{1}{3}(12) = 4 \qquad (3-7)$$

so that the tax revenue tY equals $5 \cdot 4 = 20$. Using the equivalent variation, the excess burden of the tax is the cost minus the tax revenue, $24 - 20 = 4$.