

due : Wednesday March 21 before class

Do all 5 questions. All count equally.

1. What are **all** the efficient allocations of food (a pure private good) and software (a pure public good) between two people, in an economy in which the production possibility frontier has the equation

$$X + Z = 120$$

where X is the aggregate quantity produced of food and Z is the aggregate quantity produced of software, and in which the two people have preferences represented by the utility functions

$$U^1 = \ln x_1 + \ln z_1$$

$$U^2 = \ln x_2 + 2 \ln z_2$$

where x_1 , x_2 , z_1 and z_2 are the quantities consumed by the two people of food and software (and where \ln represents the natural logarithm)?

2. Solve for the *Lindahl equilibrium* — that is the allocation at which the vertical sum of people's demand curves for the public good equals the marginal cost of the public good — if people's preferences were those defined in question# 1, and if each person had an income of 60.

3. Suppose that a government tries to implement Lindahl pricing by asking each of N people the shape of her or his inverse demand curve for some public good.

Suppose as well that the marginal cost of the public good is known to everyone, and known to equal some constant c .

So the government promises that after each person has reported a function $p_i(Z)$, expressing her or his marginal willingness to pay p_i as a function of the level Z of public good provision, then the government will provide a public good level Z^* , satisfying the efficiency condition

$$p_1(Z^*) + p_2(Z^*) + \cdots + p_N(Z^*) = c$$

and then will charge each person her or his Lindahl prices, so that person i will pay taxes of

$$p_i(Z^*)Z^*$$

If person i 's true inverse demand function for the public good is $\phi_i(Z)$, and if she is acting in her own self-interest, what sort of demand curve should she report?

4. A village has 300 families, and no hockey arena. It would cost \$210,000 to build a hockey arena. The total value to family i is v_i ; a family with valuation v_i would be indifferent between having no arena, and having the arena and paying v_i dollars.

There are 100 families for whom v_i is \$2000.

There are 100 families for whom v_i is \$1000.

There are 100 families for whom v_i is \$100.

A local organization has set up a campaign to build an arena, and is asking for contributions. It has made the following pledge : if the organization receives less than \$210,000 in contributions, every contributor will get her or his money back ; if the organization receives \$210,000 or more in contributions, then the arena will be built, and any extra money it receives (over and above \$210,000) will be returned, divided equally among all the families in the village.

If the villagers act in their own self-interest, what is the likely outcome of the campaign?

5. Now suppose that instead of the local campaign described in question #4, the village government asked each family : “What is the arena worth to you?” (that is, “what is your v_i ?”) and promised to build the arena if (and only if) the sum of people’s reported valuations exceeded the cost of the arena.

Suppose as well that each family would pay an equal share of the cost of the arena — that is, \$700 — if the arena were built.

The government does not know any family’s true valuation, which is why it is asking them.

(The villagers’ true valuations are those described in question #4.)

If the villagers act in their own self-interest, what answers will they give to the government’s questions?