AS/ECON 4080 Answers to Assignment 2 April 2001

1.(a) The mean income in the country is $\bar{y}$. Given the expression for the tax base, that means that the tax base per person in the country will be $(1-\frac{1}{2}t)\bar{y}$ if the tax rate is t, so that the tax revenue collected per person will be

$$t(1-\frac{1}{2}t)\bar{y}$$

What a person of income y receives is her net of tax income, plus her share of the tax revenue, or

$$(1-t)y + t(1-\frac{1}{2}t)\bar{y}$$

(b) People care about what they receive, $R(t) = (1-t)y + t(1-\frac{1}{2}t)\bar{y}$. How does this amount vary with the tax rate t?

$$R'(t) = -y + \bar{y} - t\bar{y}$$

and

$$R''(t) = -\bar{y}$$

These expressions mean that a person's overall income $R(t)$ is a concave function of the tax rate t. $R(t)$ has a single peak, at

$$t^*(y) \equiv \frac{\bar{y}-y}{\bar{y}}$$

is increasing in t for $t < t^*(y)$ and is decreasing in t for $t > t^*(y)$.

2. Since people's preferences are single–peaked, the tax rate chosen will be the median of the people's preferred tax rates. From the answer to 1(b), a person's preferred tax rate $t^*(y)$ is a decreasing function of her income. Therefore, ordering people in order of their income is the same thing as ordering them in (reverse) order of their preferred tax rate. The person with the median preferred tax rate will be the person of median income. From the answer to 1(b), the preferred tax rate of the person of median income is

$$t^*(\tilde{y}) = \frac{\bar{y}-\tilde{y}}{\bar{y}} = 1 - \tilde{y}/\bar{y}$$

which is the tax rate which will be selected under pairwise majority rule. As long as the median income is less than the mean income, which is true in virtually all countries, this preferred tax rate will be positive.

3. If a country had complete equality of income, then the median income would equal the mean, and the tax rate chosen in that country (in the model of questions 1 and 2) would be 0. Income inequality, and the usual shape for the income distribution, implies that the median income is less than the mean, so that the tax rate chosen would be positive.

So the model tends to suggest that, the more inequality there is in a country, the greater will be the tax rate chosen. To the extent that high levels of income taxation tend to slow economic growth, this suggestion is consistent with the macroeconomists' finding.

(A greater degree of income inequality is not exactly the same thing as the ratio $\tilde{y}/\bar{y}$ being smaller. But it is true that an increased share of income going to the very richest people will mean a lower value for this variable : increases in the income of the richest people in a country will increase $\bar{y}$ and leave unchanged $\tilde{y}$.)

4. Each type of person has a preferred level of expenditure on education, and a preferred level of expenditure on health. Given that

$$x_i = 30 - E - H$$

utility of a type–1 person can be written in terms of expenditure on health and education as

$$U^1 = 30 - E - H + 2E + 3H - \frac{1}{2}E^2 - \frac{1}{2}H^2 = 30 + E + 2H - \frac{1}{2}E^2 - \frac{1}{2}H^2$$

Differentiating this expression with respect to E, the change in the utility of a type–1 person as education expenditure changes (with the cost of the changes paid for equally by all people) is

$$1 - E$$

so that this person has single–peaked preferences over education expenditure, rising with E up to her preferred level $E_1^* = 1$, and then falling.

Similarly, such a person has single–peaked preferences over health expenditure, with a preferred level of $H_1^* = 2$.

Analogously, the other two types of people have single–peaked preferences over E and over H (when people share the costs of health and education equally), with preferred levels of

$$E_2^* = 3 \quad H_2^* = 1$$

$$E_3^* = 2 \quad H_3^* = 3$$

respectively.

The median voter theorem then says that each committee, the education committee and the health committee, will have a policy which can defeat all others in a pairwise vote. These winning policies are the median of the preferred levels of education and health expenditure : $E^m = 2$ and $H^m = 2$.

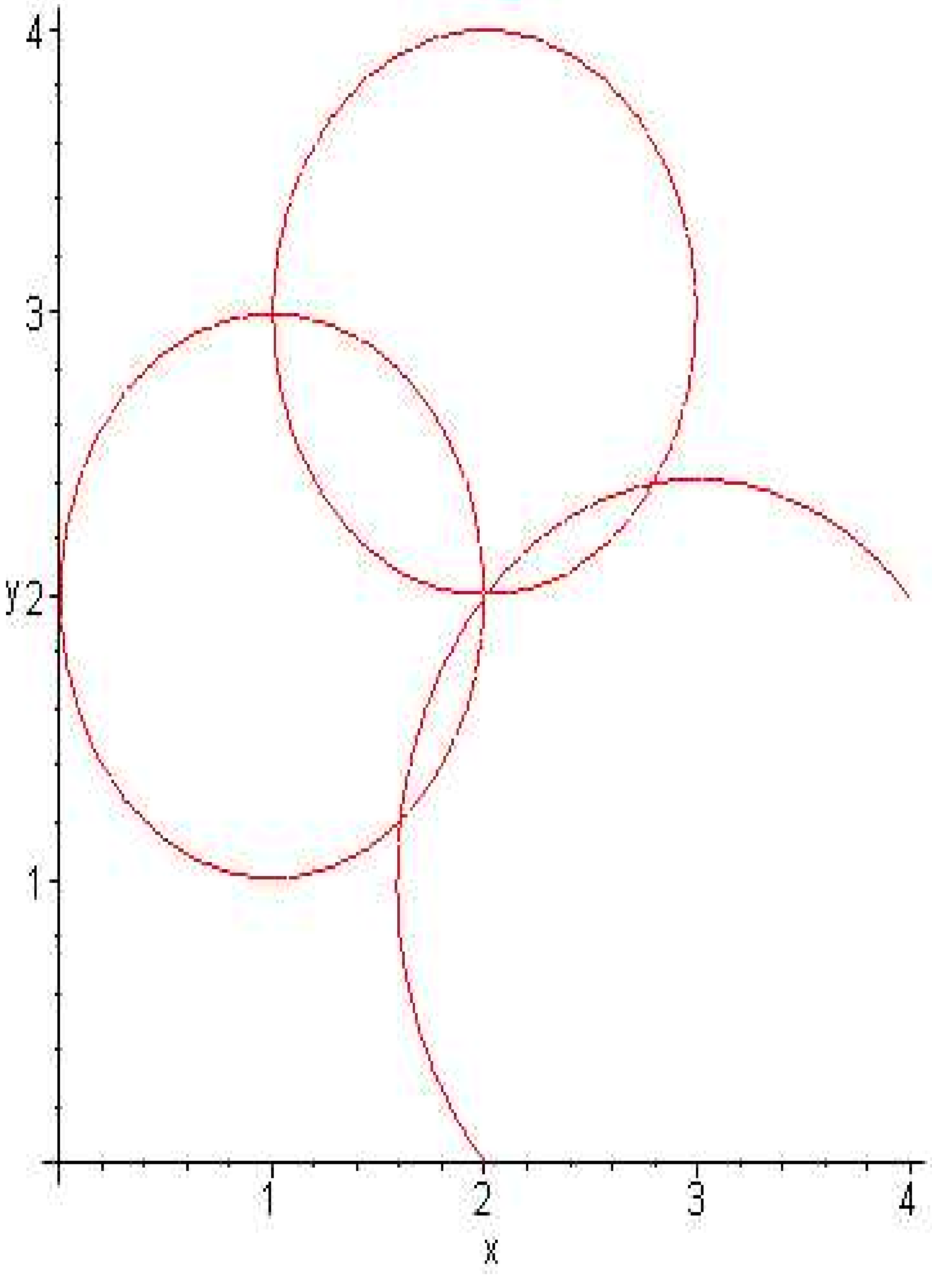
(Note that the identity of the median voters are different in the two committees. Nonetheless, two–thirds of the voters in each committee prefer a per capita spending level of 2 or more, and two–thirds prefer a per capita spending level of 2 or less.)

5. When E and H are chosen simultaneously, the alternatives cannot be arranged in a line. In this case there is no median preferred policy, and there is no single expenditure policy (H, E) which will defeat all others in a pairwise vote.

The only possible choice which might defeat all other alternatives in a pairwise vote is $E = H = 2$, because, for example, if $E > 2$, then both type–1 and type–3 people would vote for a change in E to 2, leaving H unchanged.

Even though $(2, 2)$ is the result of each committee voting separately on education and health expenditure, it can be defeated in a pairwise vote, if simultaneous changes in both H and E are allowed. For example, both type–1 people and type–3 people get higher utility from $E = 1.5$, $H = 2.5$ than from $H = E = 2$. ($H = E = 2$ gives utility of 32 to type–1 people and 36 to type–3 people ; $E = 1.5$, $H = 2.5$ gives utility of 32.25 to type–1 people and 36.25 to type–3 people.)

One way of showing that there is no policy which defeats any other is to draw indifference curves for the voters, in E–H space : that is, combinations of E and H which give the voter the same utility, given that the cost of the health and education is to be split equally among the people. For type–1 people, these are "sort of" circular curves (ellipses, actually), centred around the type's preferred combination $(1, 2)$. From $E = 2$, $H = 2$, each type would like to move towards her or his preferred combination. In the diagram, a move "up and to the left" (decreasing E and increasing E) will move both type–1 and type–3 voters to higher indifference curves, which is why $(1.5, 2.5)$ defeats $(2, 2)$.



4
3
y2
1
1
2
3
4
x