AS/ECON 4080 ASSIGNMENT 1

Due : Wednesday January 30 10:30 a.m.

Do all 5 questions. All count equally.

1. What are all the efficient allocations in the following two–good, two–person economy?

Good X is a pure private good, and good Z is a pure public good. The economy's production possibility frontier has the equation :

$$X + Z \leq 102$$

Person 1's preferences can be represented by the utility function

$$U^1(x_1, z_1) = x_1 + 2 \ln z_1$$

and person 2's by the utility function

$$U^2(x_2, z_2) = x_2 + 10 \ln z_2$$

where x_i is person i's consumption of the private good, and z_i is person i's consumption of the public good.

2. What are all the efficient allocations for an economy which is exactly the same as the economy in question #1, except that person 2's utility function is now

$$U^2(x_2, z_2) = x_2 z_2$$

(The preferences of person 1 and the production technology remain exactly as in question # 1.)?

3. Suppose that a good is non–rival, but it also is perfectly excludable. Suppose that there are N identical people in the country, and the benefit each of these people gets from having Q units available of the non–rival good is $B(Q)$, with $B'(Q) > 0$ and $B''(Q) < 0$. (That is, $B(Q)$ dollars is the amount each person would be willing to pay to get to consume a quantity Q of the non–rival good.) The total cost of producing the non–rival good is $C_0 + C(Q)$, with $C'(Q) > 0$ and $C''(Q) > 0$, where the fixed cost $C_0 > 0$.

The government plans to charge a *two–part tariff* for the non–rival good. That is, it will charge each person a flat fee F to use the non–rival good, as well as a user charge of u per unit used. (So a person consuming $q \leq Q$ units of the non–rival good would pay a total charge of $F + uq$.)

The revenues collected from fees must equal the cost of providing the public good. What level of provision Q should the government choose, and what fee structure?

4. Public transit is easily excludable, but may or may not be non–rival, depending on how congested it is.

Suppose that a public transit authority has a given number of buses, which are capable of providing T trips per hour. (T is the total number of *people–trips.* That is, T people can each take one trip per hour, or $T/2$ people each take 2 trips per hour....) The authority cannot change this capacity.

There are N identical people in the city. Each person's total benefit from being able to take t trips per hour during rush hour is $B_r(t)$, with $B_r'(T/N) > 0$ and $B_r''(t) < 0$ (for all t). Outside of rush hour, each person's total benefit from being able to take t trips per hour is $B_0(t)$, with $B_0'(t) > 0$ for all $t < \bar{t}$, $B_0''(t) < 0$ for all $t < \bar{t}$ and $B_0'(t) = 0$ for all $t \geq \bar{t}$, where $N\bar{t} < T$.

The city needs to set bus fares : f_r for each trip taken in rush hour, and f_0 for each trip taken out of rush hour. The total costs of running the transit system are C, which do not depend at all on the number of trips people take. The city can also levy head taxes of h per person, if it needs to, to help cover the cost of running the transit system.

What levels of f_r, f_0, and h should the city choose?

5. Re–do question #4 if, in addition to the costs C mentioned there, the city must also incur operating costs of c per trip taken (whenever the trip is taken). (Assume all the other data are as in question #4.)