# AS/ECON 4080 Assignment 1 <br> due : before class Wednesday January 282004 

Do all 5 questions. Each is worth $20 \%$.

1. What are all the efficient allocations in the following two-good, two-person economy?

Good $X$ is a pure private good, and good $Z$ is a pure public good. The economy's production possibility frontier has the equation :

$$
X+Z \leq 120
$$

Person 1's preferences can be represented by the utility function

$$
U^{1}\left(x_{1}, z_{1}\right)=x_{1}+10 \ln z_{1}
$$

and person 2's by the utility function

$$
U^{2}\left(x_{2}, z_{2}\right)=x_{2}+30 \ln z_{2}
$$

where $x_{i}$ is person $i$ 's consumption of the private good, and $z_{i}$ is person $i$ 's consumption of the public good.
2. What are all the efficient allocations for an economy which is exactly the same as the economy in question \#1, except that person 1's utility function is now

$$
U^{1}\left(x_{1}, z_{1}\right)=\ln x_{1}+\ln z_{1}
$$

if the preferences of person 2 and the production technology remain exactly as in question $\# 1$ ?
3. Suppose person 1's preferences could be represented by the utility function

$$
U^{1}=\ln x_{1}+a \ln z_{1}
$$

for some positive number $a$. If the person had an exogenous income $Y_{1}$, and were able to buy the private good and the public good in competitive markets, at the prices $p_{x}$ and $p_{z}$ respectively, derive an expression for the person's demand curve for the public good.
4. Suppose that the two people had the preferences listed in question $\# 2$ ( that is $U^{1}\left(x_{1}, z_{1}\right)=\ln x_{1}+\ln z_{1}$ and $U^{2}\left(x_{2}, z_{2}\right)=x_{2}+30 \ln z_{2}$ ), that the production technology was the same as in questions $\# 1$ or $\# 2$, but that a government was now trying to finance its public sector using Lindahl taxation ( also known as benefit taxation ). If person 1's income ( measured in units of the private good ) was 30 , and if person 2 's income was 90 , what would the Lindahl taxes be, and what quantity of the public good would be provided?
5. Suppose that preferences, technology, and the incomes of the two people were the same as in the previous question. However, now the public good will be financed simply by splitting the cost between the two people. If the total cost of a public good level was financed this way, what quantity of the public good would person 1 like to see provided? What quantity of the public good would person 2 like to see provided?

