AS/ECON 4080 ASSIGNMENT 1

Due : Wednesday January 26 2005 : before class

Do all 5 questions. All count equally.

1. What are all the efficient allocations in the following two–good, two–person economy?

Good X is a pure private good, and good Z is a pure public good. The economy's production possibility frontier has the equation :

$$X^2 + Z^2 = 180$$

where X and Z are the total quantities produced of the private good and of the public good, respectively.

Person 1's preferences can be represented by the utility function

$$U^1(x_1, z_1) = \ln x_1 + 4 \ln z_1$$

and person 2's by the utility function

$$U^2(x_2, z_2) = \ln x_2 + 4 \ln z_2$$

where x_i is person i's consumption of the private good, and z_i is person i's consumption of the public good.

2. What are all the efficient allocations in the following two–good, two–person economy?

Good X is a pure private good, and good Z is a pure public good. The economy's production possibility frontier has the equation :

$$X + Z = 36$$

Person 1's preferences can be represented by the utility function

$$U^1(x_1, z_1) = 2 \ln x_1 + 3 \ln z_1$$

and person 2's by the utility function

$$U^2(x_2, z_2) = 2 \ln x_2 + \ln z_2$$

where x_i is person i's consumption of the private good, and z_i is person i's consumption of the public good.

3. If person 1 and person 2 had preferences which could be represented by the utility functions

$$U^1(x_1, z_1) = (1 - \alpha) \ln x_1 + \alpha \ln z_1$$

$$U^2(x_2, z_2) = (1 - \beta) \ln x_2 + \beta \ln z_2$$

where x_i was person i's consumption of a pure private good, and z_i was person i's consumption of a pure public good, and where α and β were positive numbers (which are less than 1), how would a transfer of \$1000 from person 1 to person 2 affect the overall supply of the public good, if the public good supply was determined using the Lindahl pricing rule, if the production possibility curve were a straight line?

4. A district contains N identical people. Each of them has preferences represented by a utility function

$$U(x, Z, m) = x + [12m - m^2]Z$$

where x is each person's consumption of a pure private good, Z is the size of the park in the district, and m is the number of times each person chooses to visit the park. The price of the private good is fixed at \$1 per unit.

The park is perfectly non–rival, but it is also perfectly excludable. The government can charge an admission fee of p per trip to the park.

The total cost of building and maintaining a park of size Z is Z^2. The government can also levy a head tax of T on each person. So the government must collect enough revenue, from this head tax, and from admission charges to the park, to cover the cost Z^2 of building and maintaining the park.

What admission fee should the government charge to the park? What size of park should it choose to build?

5. Suppose that each person's marginal benefit from one more trip on some highway depends on the number of trips that she takes, as well as on the number of other drivers who are on the road.

Suppose as well that this marginal benefit decreases with the number of trips the driver takes. It does not depend on the number of other drivers on the road, unless the total number O of other drivers on the road is $\bar{O}$ or greater, where $\bar{O} > 0$. If $O > \bar{O}$, then the person's marginal benefit from one more trip on the road decreases with the number O of other drivers on the road, and decreases at a rate which increases with O. (That is,

$$\frac{\partial MB}{\partial O} < 0 \quad \text{if} \quad O > \bar{O}$$

where MB is the driver's marginal benefit from one more trip.)

If the highway has already been built and paid for, would it be efficient to charge a toll for each trip on the highway? If so, what is the efficient toll? If not, why not?