due : Wed. March 16, before class

All 5 questions count equally.

1. Suppose that a person's preferences over private consumption $x$, and the level of public expenditure $z$ could be represented by the utility function

$$
u(x, z)=1000-\frac{1}{x}-\frac{a}{z}
$$

where $a>0$ was some positive constant.
If the person's before-tax income were $y$, if public expenditure were to be financed by a proportional income tax, and if this person's share of the total income was $s$, how would she rank the different possible levels of public expenditure, knowing that they will be financed by the income tax?

Are her preferences over public expenditures single-peaked? Explain briefly.
2. For the person described in question \#1, what is her most-preferred level of public expenditure? How does it vary with the parameter $a$ in her utility function?
3. If there were many voters, each with the preferences described in question $\# 1$, and all with the same preference parameter $a$, how would voters' preferred levels of public expenditure $z$ vary with their income level $y$, if a person's share $s$ of the taxes was proportional to her income $y$ ?

## continued over

4. Three voters are deciding on public expenditure on police services and on education. They must each pay an equal share (one third) of the cost of each public expenditure category.

Each person has the same income, 600. (All variables are measured in hundreds of dollars).
If $x_{i}$ is person $i$ 's expenditure on private consumption, $Y$ is expenditure on police services, and $Z$ is expenditure on education, then the three people have preferences represented by the following utility functions:

$$
\begin{gathered}
u^{1}\left(x_{1}, Y, Z\right)=x_{1}+4 \ln Y+\ln Z \\
u^{2}\left(x_{2}, Y, Z\right)=x_{2}+2 \ln Y+4 \ln Z \\
u^{3}\left(x_{3}, Y, Z\right)=x_{3}+6 \ln Y+6 \ln Z
\end{gathered}
$$

What levels of expenditure would be chosen, under pairwise majority rule, if they were required to vote separately on each expenditure category, choosing police expenditure using pairwise majority rule in one vote, and choosing education expenditure in another vote?
5. If the people have the same preferences and incomes as in the previous question $\# 4$, and if they each must pay for one-third of any public expenditure, what level of expenditure on police services and on education would they choose, if they used pairwise majority rule, but voted separately on two issues : the aggregate amount of public expenditure (on police and education) together in one vote, and what share of total expenditure should go to education in a separate vote?

