AS/ECON 4080 ASSIGNMENT 1

Due : Wednesday January 25 2005 : before class

Do all 5 questions. All count equally.

1. What are all the efficient allocations in the following two–good, two–person economy?

Good X is a pure private good, and good Z is a pure public good. The economy's production possibility frontier has the equation :

$$X + Z^2 = 360$$

where X and Z are the total quantities produced of the private good and of the public good, respectively.

Person 1's preferences can be represented by the utility function

$$U^1(x_1, z_1) = x_1 + 50z_1 - 2(z_1)^2$$

and person 2's by the utility function

$$U^2(x_2, z_2) = x_2 + 30z_2 - (z_2)^2$$

where x_i is person i's consumption of the private good, and z_i is person i's consumption of the public good.

2. What are all the efficient allocations in the following two–good, two–person economy?

Good X is a pure private good, and good Z is a pure public good. The economy's production possibility frontier has the equation :

$$X + Z = 60$$

Person 1's preferences can be represented by the utility function

$$U^1(x_1, z_1) = 2\ln x_1 + 8\ln z_1$$

and person 2's by the utility function

$$U^2(x_2, z_2) = \ln x_2 + \ln z_2$$

where x_i is person i's consumption of the private good, and z_i is person i's consumption of the public good.

3. What would the Lindahl equilibrium allocation be for the economy described in question #1 above, if the price of the private good were 1, if person #1's income were 240, and if person #2's income were 120?

4. A city contains N identical people, each of whose preferences can be represented by the utility function

$$U(x, z) = x + a \ln z$$

where x is the person's consumption of a private good, z is the number of trips the person takes per day on a highway, and a will be defined below.

The parameter a measures the speed of traffic flow on the road, and is defined as

$$a = \sqrt{\frac{K}{Nz}}$$

where K is the size of the road. The total cost of a road of size K is cK.

If the city government could control how many trips each person takes per day on the highway z, and the size K of the road, what would be the efficient choices of z and K?

5. In the city described in question #4 above, suppose that the city government could not control directly the number of trips each person could take on the road, but could set a toll p for each trip taken by any person.

What toll should it charge? Would toll revenue cover the cost of the road?