Public Choice : $(a)$ Social Choice Rules

Social choice rules are rules under which a group of people can make a choice from a set of **alternatives**. The set of alternatives might consist of different candidates for some official position, different levels of some tax rate, different routes for a proposed new subway, different nicknames for a school's athletic teams, different proposals for a constitution, different candidates for the "Best Actor" award, and so on.

What these sets of alternatives have in common are that they are **mutually exclusive**, and that each person in the group has **preferences** over the set of alternatives. "Mutually exclusive" means that we must choose only one of the alternatives. It is this mutual exclusivity that makes it a choice problem : accepting one alternative means rejecting all of the others. People, it is assumed, are able to **rank** the alternatives, from most preferred to least preferred. The social choice problem can be a problem because different people may rank the alternatives differently.

Often, we want more than just a single choice from the set of alternatives. We may want a **complete ranking** of all the alternatives. For example, the city may not know how many subway routes it can afford. It may be able to afford to build 1, or 2, or maybe 3 routes, but it does not yet know exactly how many. So it needs to come up with an ordered list, so that it will build the route listed at the top of the list if it can only afford one route, but will build the top three routes on the list if it can afford to build three routes.

A **social choice rule** is a procedure for making a choice from the set of alternatives. Different institutions use different rules in making choices. Here are a few examples.

Example 1 : **plurality voting** : This is the usual rule for elections, in which the set of alternatives is a group of candidates, running for mayor, or member of parliament, or a position on student council. Some body of electors (for example, all residents of the city, over the age of nineteen, or all students enrolled in a particular college) has to choose from this set of candidates. So each of these voters gets to mark a ballot, and can vote for only one of the candidates. What is the social choice rule here, for determining which candidate is chosen? The candidate chosen is the candidate with the most votes. ( If two candidates were tied with the most votes, then maybe we would decide the outcome by flipping a coin. ) This rule also gives a complete ordering of the candidates : the candidates with the most votes is ranked first, then the candidate with the next–most votes is ranked second, and so on.

Example 2 : **plurality with a runoff election** : In some American states ( and some Latin American and European countries ), candidates are elected using a procedure which may involve two stages of voting. In the first stage, each voter votes for a single candidate. The votes are counted, and if any one candidate gets more than 50 percent of the votes, then that candidate is chosen. However, if no single candidate gets a majority of the vote, then a second "runoff" election is held. Only the two candidates with the most votes in the first stage are listed on the ballot for this second stage. Each person votes for one of the two candidates on this second ballot, and the candidate who gets the most votes in this second stage is then chosen.

Example $2a$ : **plurality with many runoffs** : We could run an election with many stages, more than the two stages in the previous example. In any stage, if a candidate gets a majority of the votes, then she is chosen. If no candidate has a majority at this stage, then the candidate with the fewest votes is crossed off the ballot, and a new stage is conducted, with one candidate less on the ballot. These stages continue until one candidate gets a majority of the votes. This procedure is often used at political conventions, in which national political parties choose a leader.

Example 3 : **pairwise majority rule** : Under plurality ( and in the first stage of plurality with a runoff ), **all** the alternatives are listed on the ballot. That is not how legislatures ( such as city councils, or the House of Commons ) operate. Legislatures have to choose from a set of mutually exclusive alternatives : what will be the city's property tax rate? what project should be put on the land formerly occupied by Downsview air base? what limits should be placed on where homeless people can sleep? In each case, there are many possible proposals, and the legislature has to choose one of them to be the law.

What happens in a legislature is that one of the legislators (that is, one of the city councillors, or one of the members of parliament) proposes an alternative. All the legislators then vote, either for the alternative which has just been proposed, or against it. So voting against the proposed alternative is voting for whatever policy has been in place so far ( the **status quo** ). If the proposed alternative gets more "for" votes than "against", then it is set in place. It becomes the new status quo. But, after this vote, other legislators can propose different new alternatives. This process may have many stages, perhaps an infinite number, depending on how many times legislators are allowed to make proposals. But at each stage, legislators are choosing between **pairs** of alternative : either a "yes" vote in favour of the new proposal or a "no" vote, which is a vote in favour of the status quo.

Example 4 : **Borda count** : This method is often used in voting for "best player" awards in sports, or to construct a ranking of the best teams. The ranking may be chosen by a group of journalists, or coaches, or experts. But here each voter does not vote for a single candidate, but for, for example, her 5 most preferred candidates. She provides a list, of her 5 preferred alternatives, ordered from best to 5th–best. Then scores are assigned to alternative : each first–place ranking by a voter gives 5 points to that alternative, each second–place ranking gets 4 points, and so on. All the voters' scores are added up. The alternative with the highest total score is chosen, and the alternatives are ranked in order of the total scores they get.

Example $4a$ : more **Borda count** examples : Voters don't need to list their top 5 choices. We might require them to list their top ten alternatives, giving 10 points for each first–place ranking, 9 for each second–place ranking, and so on. Or we could change the scoring. With lists containing each person's top 4 alternatives, we could give 10 points for each first–place ranking, 5 points for each second–place ranking, 3 for each third–place ranking, and 1 for each fourth–place ranking.

Example 5 : **unanimity** As under pairwise majority rule, this rule ranks *pairs* of alternatives, rather than the whole set of alternatives. Alternative $A$ is ranked above alternative $B$ only if **every** person votes for $A$ over alternative $B$. So, under unanimity, if 98 out of 99 people voted for $A$ over

$B$, and the remaining person voted for $B$, then $A$ would **not** be ranked above $B$, since it is not preferred unanimously. $B$ would certainly not be ranked above $A$ either, in this example. Under the unanimity rule, if one alternative $A$ is not ranked as better than $B$ by **all** the voters, then we simply cannot rank the alternatives $A$ and $B$.

An example

Suppose that there were 5 alternatives, $A$, $B$, $C$, $D$ and $E$, and 7 voters. The rankings of the voters are listed in the following table :

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| $A$ | $B$ | $A$ | $C$ | $C$ | $A$ | $E$ |
| $B$ | $C$ | $B$ | $A$ | $A$ | $B$ | $B$ |
| $C$ | $D$ | $E$ | $B$ | $B$ | $E$ | $C$ |
| $D$ | $E$ | $C$ | $E$ | $D$ | $C$ | $D$ |
| $E$ | $A$ | $D$ | $D$ | $E$ | $D$ | $A$ |

In the above table, each column represents the ranking of the alternatives by a different voter. The numbers (1 through 7) are the names of the voters. The letters ($A$ through $E$) are the different alternatives. The 6th column, for example, indicates that alternative $A$ is voter # 6's most preferred choice, followed by $B$ followed by $E$, followed by $C$, followed by $D$.

It will be assumed (first) that people vote **sincerely**. That means that each voter would vote for the candidate she really does prefer, under majority rule, and will write down the alternatives in the order that she really does rank them, if a Borda count is being used. This is a strong assumption. As will be seen later, a person often may have strong incentives to vote **strategically**, for example to vote for an alternative which is not really her most preferred in a plurality election.

But under the assumption that people vote sincerely, the winner under plurality voting would be alternative $A$ in this example. Under plurality voting, if people vote sincerely, each person will mark an "X" beside the alternative that she ranks as most preferred : here people #1, 3 and 6 will vote for $A$, person # 2 will vote for $B$, people # 4 and 5 will vote for $C$, and person #7 will vote for $E$. So $A$ gets 3 votes, $C$ 2 votes, $B$ and $E$ 1 vote each, and $D$ gets no votes.

However, $A$ does not get a majority of the votes under plurality voting. If the rule were different, if it were plurality with a run–off, then there would be a second–stage ballot, a contest between $A$ and the runner–up $C$ from the first stage. In this second stage, voters 2 and 7 would vote for $C$, since they each rank $C$ above $A$ ( and because the other three alternatives are no longer on the ballot ). So $C$ would be chosen if the rule were plurality with a runoff. In other words, changing the social choice rule has changed the outcome chosen.

If a third rule were chosen, Borda count, with 5 for first place, 4 for second place, 3 for third place, 2 for second place and 1 for last place, there would be a different result. Under this Borda count, $A$ gets 25 points : 15 for 3 first–place rankings, plus 8 for two second–place rankings, and 2 for two fifth–place rankings. $C$ gets only 24 points. But alternative $B$ gets 27 points ( 5 for its one first–place ranking, 16 for its 4 second–place rankings, and 6 for its two third–place rankings ). So alternative $B$ is the winner under this Borda count rule.

Changing the Borda count rule to 10–5–3–1–0 ( that is, 10 for a first–place ranking, 5 for a second–place ranking, etc. ) would change the outcome : you should check that, under this social choice rule, alternative $A$ would be chosen.

What about pairwise majority rule? $C$ defeats $A$ in a pairwise vote if people vote sincerely : people # 2, 4, 5 and 7 rank $C$ above $A$, while people #1, 3 and 6 prefer $A$ to $C$. So alternative $C$ would defeat alternative $A$ by a margin of 4 to 3. Alternative $B$ will defeat alternative $C$ under pairwise majority rule : people # 1, 2, 6, and 7 all rank $B$ above $C$, people # 3, 4 and 5 rank $C$ above $B$, so that $B$ defeats $C$ by a vote of 4 to 3. However, alternative $A$ actually can beat $B$ in a pairwise election : people # 1, 3, 5 and 6 all rank $A$ above $B$, people #2, 4 and 7 rank alternative $B$ above alternative $A$, so that $A$ defeats $B$ by a vote of 4 to 3. * There are **cycles** under pairwise voting. If voters are allowed to keep introducing new proposals, then the outcome will keep switching (if people vote sincerely), since there is no single alternative which can defeat each other alternative in a pairwise vote.

How could this endless cycling be avoided? Perhaps the rules of the legislature might restrict the number of proposals. The rules might also specify the order in which new proposals were introduced. One rule might start with a pairwise vote between $A$ and $B$, then have alternative $C$ paired against the winner of that first vote, $D$ against the winner of that second vote vote, and finally $E$ against the winner of the third vote. If people vote sincerely, then $A$ wins the first vote against $C$, but loses the second vote against $C$. $C$ then defeats $D$ and $E$ in pairwise votes, and so is the overall winner.

But if the order of votes is different, the outcome may be different. If the order is reversed — first $E$ versus $D$, then $C$ versus the winner of the first vote, then $B$ versus the previous winner, then $A$ versus the previous winner — a different process occurs. $E$ wins the first vote against $D$, but then loses to $C$. $B$ defeats $C$ and then $A$ defeats $B$ so that $A$ is the winner. In this example, under pairwise majority rule with a fixed agenda, the order of the agenda influences the outcome, if people vote sincerely.

Finally, what does unanimity provide for social choice? Not much, in this example. All 7 people rank $C$ above $D$, so that the unanimity rule says that the group ranks $C$ above $D$. $B$ is also preferred unanimously to $D$. But that's it. There are 18 other pairs of alternatives ( $A$ versus $B$, $A$ versus $C$, and so on ), and in **none** of them is the ranking unanimous. In each of these 18 other comparisons, there is at least one person who ranks each alternative higher. So requiring unanimous agreement here means that an overall ranking cannot be agreed upon.

So a few problems arise with the social choice rules, as used in this example. Different rules give different outcomes. And some rules may give no outcome at all.

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* You can also check that here alternative $E$ defeats $D$, and any one of the alternatives $A$, $B$ or $C$ would defeat either of $D$ or $E$.

The outcomes were all calculated under the assumption that people voted sincerely. Is that likely? Consider person 7, the only person who really likes alternative $E$. Under **plurality voting**, she votes for $E$, which gets no other votes. The outcome, under plurality voting, is that $A$ is chosen, person 7's least favourite alternative. If 7 notices that most of the other people rank $A$ or $C$ first, then she might realize that $E$ has no chance : the vote is really a race between $A$ and $C$. So maybe she should vote for $C$, her third–favourite alternative, instead of her preferred alternative $E$. If she switches her vote from $E$ to $C$, and if everyone else votes sincerely, now $A$ and $C$ both get 3 first–place votes. So $C$ will get chosen half the time, if a coin flip is used to break ties. Voting strategically has improved the outcome for person 7. Instead of getting her least–preferred alternative ($A$) for sure, she gets a 50–percent chance at a slightly better alternative.