Externalities : (b) Person–to–Person

In this section, the algebra of efficiency (and inefficiency) is derived for a simple two–person economy with an externality. The algebra will look very similar to the algebra of public good provision in a 2–person economy. This is not a coincidence. In many respects the negative externality which will be analyzed here acts just like a **public bad**. With public goods, person #2's consumption of the public good did not affect the opportunities of person #1 to consume the good, and neither person could be excluded from enjoying the benefits of the public good. Here, person #2's "forced" consumption of the bad (to him) will not affect person #1's opportunities to consume the good (to her), and person #2 cannot be excluded from suffering the damage from the public bad.

So consider a two–person, two–good world (much like the one in the second lecture "Efficiency", in the section on public goods). The two goods are an "ordinary" good X, and another good Z which produces an externality. The production technology of the economy will be described by a production possibility frontier, with equation

$$X = F(Z) \tag{1}$$

where $F'(Z) < 0$, and $F''(Z) \leq 0$. The marginal rate of transformation for this technology is defined, as in the public goods example, by

$$MRT \equiv -F'(Z) \tag{2}$$

So $-F'(Z)$ measures how much production of food (good X) would have to fall, if we were to increase cigarette (good Z) production by one unit. So it measures the opportunity cost of good Z.

The two people are called person #1 and person #2. Person 1's preferences can be represented by a utility function

$$U^1(x_1, z_1) \qquad U^1_x > 0 \quad U^1_z > 0 \tag{3}$$

where x_1 and z_1 are her consumption of the two goods X and Z respectively, and where U^1_x and U^1_z are her marginal utilities of consumption of the two goods. So, for person 1, both goods are ordinary goods : the more she consumes of each, the better off she is. As usual, the rate at which she is willing to substitute one good for another, her marginal rate of substitution can be defined as

$$MRS^1 \equiv \frac{U^1_z}{U^1_x} \tag{4}$$

So MRS^1 is how much person #1 is willing to pay for another cigarette (good Z) measured in units of food (good X).

On the other hand, person #2 does not like good Z. She certainly does not want to buy any of the good on her own. She will spend all her available income on good X, since she doesn't like good Z.

However, there is a negative externality here. Person #2 cannot avoid the effects of person #1's consumption of good Z. In particular, the more person #1 consumes of good Z, the worse off person #2 is. So person #2's preferences can be represented by the utility function

$$U^2(x_2, z_1) \qquad U_x^2 > 0 \quad U_z^2 < 0 \tag{5}$$

Equation (5) is different than the ordinary utility function in a couple of respects. First of all, person #2's utility depends not on his own consumption of good Z, but on the other person's consumption : that's a z_1, not a z_2, in equation (5). Second, person #2's utility is **decreasing** in z_1. The **marginal damage** to person #2 done by person #1's consumption of good Z, MD^2, is defined by

$$MD^2 \equiv -\frac{U_z^2}{U_x^2} \tag{6}$$

Note that MD^2 has been defined to be a **positive** number (since $U_z^2 < 0$, and since there is a minus sign in front of the expression on the right of equation (6)). MD^2 measures, in units of food (good X), the harm done to person #2 by person #1's consumption of good Z. It also measures how much food (good X) that person #2 would be willing to give up, if he could get person #1 to reduce her consumption of good Z by a small amount.

Given that there are only 2 goods, and only 2 people, and that one person doesn't even want to consume one of the goods, an allocation is a choice of quantities of food x_1 and x_2 for each of the two people, and a quantity of cigarette consumption z_1 for person #1. From equation (1),an allocation (x_1, x_2, z_1) is feasible if

$$x_1 + x_2 \leq F(z_1) \tag{7}$$

To examine efficiency, the convenient fiction of a social welfare function will be used, just as it was in the section on efficient provision of a public good. So

$$W[U^1(x_1, z_1), U^2(x_2, z_1)] \qquad W_1 > 0 \quad W_2 > 0 \tag{8}$$

is supposed to represent the way some decision maker trades off the well–being of the two people. The welfare measure defined in equation (8) is increasing in each person's utility. (W_i measures how much a small gain in person i's well–being affects overall social welfare.)

The efficiency problem is to choose an allocation (x_1, x_2, z_1) to maximize social welfare, subject to the feasibility constraint (7). Treating this constraint as an equality (we would not want to allocate less food than we have available), this constrained maximization can be solved by maximizing the Lagrangean function

$$\mathcal{L}(x_1, x_2, z_1; \lambda) \equiv W[U^1(x_1, z_1), U^2(x_2, z_1)] + \lambda[F(z_1) - x_1 - x_2] \tag{9}$$

The first–order conditions for welfare maximization are that the partial derivatives of $\mathcal{L}$ with respect to x_1, x_2 and z_1 equal zero, or

$$W_1 U_x^1 - \lambda = 0 \tag{$x1$}$$

$$W_2 U_x^2 - \lambda = 0 \tag{x2}$$

$$W_1 U_z^1 + W_2 U_z^2 = -\lambda F'(Z) \tag{z1}$$

Equation ($x1$) implies that

$$W_1 = \frac{\lambda}{U_x^1} \tag{x1'}$$

and equation ($x2$) implies that

$$W_2 = \frac{\lambda}{U_x^2} \tag{x2'}$$

so that substituting ($x1'$) and ($x2'$) into equation ($z1$) yields

$$\lambda \frac{U_z^1}{U_x^1} + \lambda \frac{U_z^2}{U_x^2} = \lambda[-F'(Z)] \tag{10}$$

which (using the definitions (2), (4) and (6) of the MRT, MRS^1 and the marginal damage MD^1) can be written

$$MRS^1 - MD^2 = MRT \tag{11}$$

or

$$MRS^1 = MRT + MD^2 \tag{12}$$

Equation (11) or (12) is very similar to the Samuelson condition : it says that, to measure the overall benefit of a little more cigarette production, we add up the marginal benefits of all people. The difference here is that the "marginal benefit" to person #2 is negative. If cigarette production is increased slightly, what happens? Three things : (i) food production must be reduced ; (ii) person #1, the only person who likes to smoke, gets to smoke more cigarettes ; (iii) person #2 is forced to consume more second–hand smoke. The three terms in equations (10), (11) and (12) represent these three effects.

Equation (12) is probably the most useful form of this efficiency condition. It says that, if an allocation is efficient, that the marginal benefit of a little more cigarette consumption by person #1 — MRS^1 — should equal the sum of the two costs imposed by the increased cigarette production : the opportunity cost MRT of reduced food production, and the marginal damage MD^2 of the increase in second–hand smoke to which person #2 is exposed.In equation (12) all three of these measures are denominated in units of the numéraire good, food (good X).

The right hand side of equation (12) is also often referred to as the **marginal social cost** of increased Z consumption by person #1.

$$MSC \equiv MRT + MD^2 \tag{13}$$

The MSC is the "true" cost of a little more Z consumption by person #1 : the cost of the cigarettes, and the cost of the second–hand smoke damage to person #2.

What happens in a market economy, in which person #1 ignores the effect of her externality? Perfect competition implies that the relative prices of goods equal the relative costs. That is, profit maximization by competitive firms implies that, in a competitive equilibrium

$$\frac{p_z}{p_x} = MRT \tag{14}$$

where p_x and p_z are the prices of the two goods. How does person #1 choose her consumption bundle, if she ignores the effect of any externalities? She has a given income m_1. She then chooses the consumption bundle (x_1, z_1) which she prefers most, from all the bundles which she can afford. That is, she maximizes her utility $U^1(x_1, z_1)$ among all the consumption bundles (x_1, z_1) for which $p_x x_1 + p_z z_1 \leq m_1$. The consumption bundle she chooses, the solution to her own utility maximization problem,is a bundle for which her indifference curve is tangent to her budget line, that is for which

$$MRS^1 = \frac{p_z}{p_x} \tag{15}$$

Equation (15) is the standard condition for consumer demand. But, if I substitute from equation (14), I get the result that, in competitive equilibrium, person #1 chooses a consumption bundle (x_1, z_1) for which

$$MRS^1 = MRT \tag{16}$$

Of course, if there were no externality, equation (16) would be (part of) the condition for Pareto optimality. But with the externality, equation (16) leads to an **inefficient** choice by person #1. Equation (16) is different than the correct efficiency condition (12), since MD^2 is left out of the right side of equation (16).

In a sense, person #1 consumes "too much" of good Z, since she takes into account only **part** of the cost of Z when she makes her consumption decisions : since she has to pay for the good, she does take into account its opportunity cost of production, the MRT. But she neglects the other part of social cost, the marginal damage done to person #2.

Now there is, typically, no single "best" allocation (x_1, x_2, z_1). Just as in the case of public goods, there is a whole family of efficient allocations, each of which satisfies condition (12). A different welfare function $W(U^1, U^2)$ will give rise to a different efficient allocation. For example, if the welfare function gave more weight to person #1's well–being, then the efficient levels of x_1 and z_1 would rise, usually,and x_2 would fall. But each efficient allocation would satisfy condition (12).

There is also, typically, no single equilibrium allocation. Person #1's choice of x_1 and x_2 depends on her income m_1. Usually an increase in her income m_1 would lead to her consuming more of both goods (if both goods were normal goods). But any equilibrium allocation will satisfy equation (16). And equation (12) can never be the same as equation (16), if person #2 is harmed by person #1's consumption of good Z.They differ by the term MD^2 which is positive.

As an example to show that there are, in general, a multiplicity of efficient allocations which satisfy the efficiency condition (12), suppose that

$$U^1(x_1, z_1) = x_1 z_1 \tag{17}$$

$$U^2(x_2, z_1) = x_2(120 - z_1) \tag{18}$$

$$F(Z) = 120 - Z \tag{19}$$

From equations (17), and (18),

$$MRS^1 = \frac{U_z^1}{U_z^1} = \frac{x_1}{z_1} \tag{20}$$

$$MD^2 = -\frac{U_z^2}{U_x^2} = \frac{x_2}{120 - z_1} \tag{21}$$

and from equation (19),

$$MRT = -F'(Z) = 1 \tag{22}$$

in this example. So the efficiency condition (12) becomes, in this example,

$$\frac{x_1}{z_1} = \frac{x_2}{120 - z_1} + 1 \tag{23}$$

Any allocation (x_1, x_2, z_1), with all three numbers non–negative, which is feasible ($x_1 + x_2 = F(z_1) = 120 - z_1$), and which satisfies equation (23), will be efficient. For example

$$x_1 = 0 \quad x_2 = 120 \quad z_1 = 0$$

$$x_1 = 18.33 \quad x_2 = 91.67 \quad z_1 = 10$$

$$x_1 = 33.33 \quad x_2 = 66.67 \quad z_1 = 20$$

$$x_1 = 45 \quad x_2 = 45 \quad z_1 = 30$$

$$x_1 = 58.33 \quad x_2 = 11.667 \quad z_1 = 50$$

are all efficient allocations in this example (since they all are feasible, and all satisfy equation (23)).

So, even though there are many efficient allocations, and many equilibrium allocations, when there is an externality which is not internalized, then the equilibrium allocations are inefficient. We often say that person #1 consumes "too much" of the externality–causing good Z. Figure 1 shows why. Given that her marginal benefit curve (MRS^1) slopes down as a function of her consumption z_1 of the externality–causing good, her equilibrium consumption level will be higher than the efficient level, at which $MRS^1 = MSC$.

This figure also may illustrate why there is not, in general, a unique efficient level of the externality–causing activity when the externality occurs between people. Person 1's marginal benefit from a little more of good Z depends on how much she consumes of other goods, as well as on her consumption of good Z. If good Z were a normal good, then if person 1's income were higher, her marginal benefit curve — her demand curve for good Z — would shift up. Conversely, if "freedom from the externality" is a normal good — that is, if the amount person 2 is willing to pay for a reduction in the negative externality goes up with his income — then the marginal damage curve for person 2 would shift down if his income were to fall. So a transfer of income from person 2 to person 1 would shift up the marginal benefit curve for person 1, and shift down the marginal damage curve for person 2, if all goods were normal. That means that the intersection of the marginal benefit curve with the marginal social cost curve in the figure would shift right : the efficient level of the externality–causing activity would go up if income were transferred from person 2 to person 1.