Externalities : (c) Firm–to–Firm

The case in which one firm's activities cause an externality which affects another firm is actually a little easier to analyze than the person–to–person case analyzed in the previous part. That is because firms do not have income effects. There will, in the case of firm–to–firm externalities, be a unique efficient level of the externality–causing activity.

However, the basic conclusions of the previous part remain true. Those two main conclusions are : (i) efficiency requires that the marginal benefit of the externality–causing activity (to the firm which is undertaking the activity) must equal the marginal social cost, which is the marginal cost to the firm of the activity **plus** the marginal damage done to other firms, and ii if the first firm ignores the externality, then the competitive equilibrium will be inefficient.

It will be assumed here that

Assumption : both firms are perfect competitors, in both the markets for the products that they sell, and in the markets for the inputs that they buy

Under perfect competition, firms' profit–maximizing behaviour is consistent with Pareto optimality — if there are no externalities.

So assume that firm produces some good, using two inputs to production, labour L and coal Z. Let $F^1(L_1, Z_1)$ denote the firm's production function, expressing the quantity of output it can produce, if it hires L_1 person–hours of labour, and uses Z_1 tons of coal. The market price of labour is w_L per hour, and the market price of coal is w_Z. Firm 1 sells its output on perfectly competitive markets, at a price of p_1 per unit of output.

So, if firm 1 hires L_1 person–hours of labour, and uses Z_1 tons of coal, then it will be able to produce $F^1(L_1, Z_1)$ units of output, which it can sell for a price of p_1 per unit. Its total profits π_1 will be

$$\pi_1 = p_1 F^1(L_1, Z_1) - w_L L_1 - w_Z Z_1 \tag{1}$$

If it ignores the externality, it will choose its input quantities so as to maximize its profit defined in equation (1). If π_1 is maximized with respect to L_1 and Z_1, then the quantities which maximize firm 1's profits are defined by

$$p_1 \frac{\partial F^1}{\partial L_1} = w_L \tag{2}$$

$$p_1 \frac{\partial F^1}{\partial Z_1} = w_Z \tag{3}$$

So equations (2) and (3) say that, to maximize profit, the competitive firm should find the input quantities such that the value of each input's marginal product equals the price of the input. In particular, the term on the left side of equation (3), $p_1 \frac{\partial F^1}{\partial Z_1}$, represents the **marginal private benefit** to firm 1 of using a little more coal : it equals the added revenue it would get from selling the added output produced by using a little more coal.

However, there is an externality here. Firm 2 is located near firm 1, and its output is affected by the amount of coal which firm 1 uses. So the output of firm 2 will be written $F^2(L_2, Z_1)$. Firm 2 uses labour as an input, and the more labour which it hires, the more output it produces. Its output is also affected — negatively — by the amount of coal being used by firm 1. So firm 2's output depends on its own labour usage (L_2), but on the coal usage of the **other** firm (Z_1).

So that's the externality : more coal usage by firm 1 will reduce the output produced by firm 2.

If firm 2 sells its output on competitive markets, at a price of p_2 per unit, then its profits π_2 can be defined as

$$p_2 F^2(L_2, Z_1) - w_L L_2 \tag{4}$$

Acting on its own, firm 2 would choose its quantity L_2 of labour to hire so as to maximize its own profits, so that its profit—maximizing hiring decision is to find the level of L_2 such that

$$p_2 \frac{\partial F^2}{\partial L_2} = w_L \tag{5}$$

But, in the absence of any legal rights, or government intervention, or negotiation, firm 2 cannot control the amount of pollution to which it is exposed. The marginal damage of the pollution can be expressed as

$$MD^2 \equiv -p_2 \frac{\partial F^2}{\partial Z_1}(L_2, Z_1) \tag{6}$$

(where the partial deriviative $\partial F^2/\partial Z_1$ is a function of the amount of input L used by firm 2, and of the amount of input Z used by firm 1). Since increases in coal usage by firm 1 **reduce** output by firm 1, $\partial F^2/\partial Z_1 < 0$, so that the marginal damage defined by expression (6) is a positive number. It represents the reduction in profits for firm 2, caused by a small increase in coal usage by firm 1.

Given that both firms are perfect competitors, an efficient choice of inputs is a choice which maximizes the **joint profits** of the two firms. That's what would be maximized if one entrepreneur bought both firms : the entrepreneur would internalize the externality. She would realize that using more coal in division #1 (the former #1) would lower profits at division #2 (the former firm #2). From equations (1) and (4), these joint profits are

$$\pi_1 + \pi_2 = p_1 F^1(L_1, Z_1) + p_2 F^2(L_2, Z_1) - w_L(L_1 + L_2) - w_Z Z_1 \tag{7}$$

Maximization of these joint profits with respect to the quantities of labour used by each firm, and with respect to the quantity of coal used by firm 1, yields the first–order conditions (where the subscripts Z and L on the left hand side of equations (8), (9) and (10) refer to the partial derivatives with respect to L_1, L_2 and Z_1)

$$p_1 F^1_L(L_1, Z_1) = w_L \tag{8}$$

$$p_2 F^2_L(L_2, Z_1) = w_L \tag{9}$$

$$p_1 F_Z^1(L_1, Z_1) + p_2 F_Z^2(L_2, Z_1) = w_Z \tag{10}$$

Given the definition (6) of the marginal damage done by pollution from the coal, equation (10) can be written

$$p_1 F_Z^1(L_1, Z_1) = w_Z + MD^2 \tag{11}$$

Equation (11) is the efficiency condition for the externality–causing activity. It pretty similar to the efficiency condition for a person–to–person externality from the previous part of this section : the marginal private benefit (to firm 1) of increased coal usage should be set equal to the marginal social cost of more coal usage ; this MSC is the sum of the opportunity cost of the coal, and the marginal damage done to the other firm.

If firm 1 ignores the externality, then its profit–maximizing decisions will be inefficient. Equation (3) is different than equation (11) : the externality–ignoring firm would take into account only the private cost w_Z of using a little more coal, and neglect the other part of marginal social cost, the damage done to the other firm.

Figure 2 depicts firm 1's marginal benefit $p_1 F_Z^1$ from coal, the cost of coal, the damage $-p_2 F_Z^2$ done to firm 2, and the efficient and equilibrium levels of coal usage. Of course figure 2 is just a re–labelled version of figure 1. Whether parties to an externality are firms or people, the efficiency condition is $MB^1 = MSC = MPC + MD^2$, where MB^1 is the marginal benefit of the externality–causing activity (to the party creating the externality), and where MPC is the marginal **private** cost of the activity. When people or firms ignore an externality, they set $MB^1 = MPC$, and undertake too much of the activity, since some of the costs (the damages to firm or person #2) are being ignored.

But with firm–to–firm externalities, there is a unique "best" level of the externality–causing activity Z. Income changes do not shift the curves in figure 2, since the firm's profit–maximizng choices do not depend on their income. Because there is a unique efficient level Z^* for the externality–causing activity, I will concentrate on firm–to–firm externalities in the following parts of this section.