Some Notation

Let

 B^i_j : size of ${\bf tax}~{\bf base}$ number j, in province number i

In the current Canadian equalization system, 5 tax bases are used. So if j were "business income taxes" (which is one of the 5 bases), and i were "New Brunswick", then B_j^i would be the total taxable income of corporations in New Brunswick. ¹ If j were "personal income tax" (another of the 5 bases used in the Canadian system), and i were Québec, then B_j^i would be the total personal income of all the residents of Québec.

Let

 P^i : population of province i

Then, in per capita terms,

$$b_j^i \equiv \frac{B_j^i}{P^i}$$

is the size of tax base j, per person, in province i. Let

 t_i^i : tax rate levied by province *i* on tax base *j*

Now if we add up over all 10 provinces, the total tax base in each category can be defined

$$B_j = \sum_{i=1}^{10} B_j^i \quad j = 1, 2, \cdots, 5$$

The tax revenue collected from tax base j in province i is

$$R^i_j \equiv t^i_j B^i_j$$

In per capita terms, that's

$$r_j^i \equiv \frac{R_j^i}{P^i} = t_j^i b_j^i$$

and for Canada as a whole, the total revenue collected (in category j) by all the provinces together is

$$R_j \equiv \sum_{i=1}^{10} R_j^i$$

¹ Actually, the corporate income tax base used for equalization also includes the net revenue of provincially–owned crown corporations, in addition to profits of privately–owned corporations in the province.

The average tax revenue collected per person, among all the provinces in category j is therefore

$$r_j \equiv \frac{R_j}{P}$$

if ${\cal P}$ is defined s the total population of Canada

$$P \equiv \sum_{i=1}^{10} P^i$$

The average tax rate among the provinces for tax base category j can then be calculated :

$$t_j \equiv \frac{R_j}{B_j} \quad j = 1, 2, \cdots, 5 \tag{1}$$

Note that t_j is **not** simply the arithmetic average of tax rates in the ten provinces. Definition (1) could also be written

$$t_j = \sum_{i=1}^{10} s_j^i t_j^i \quad j = 1, 2, \cdots, 5$$
(2)

where

$$s_j^i \equiv \frac{B_j^i}{B_j}$$

is the **share** of the total tax base j in Canada which is located in province i. The average tax base per capita for category j can be defined :

$$b_j \equiv \frac{B_j}{P} \tag{3}$$

which also can be written

$$b_j = \sum_{i=1}^{10} p^i b_j^i \tag{4}$$

where

$$p^i \equiv \frac{P^i}{P}$$

is province i's share of the total population of Canada.

An "Ideal" Equalization Programme

The "ideal" programme defined here is **not** the actual formula used to calculate equalization payments in Canada. It never was. But is a useful starting point to see the basic philosophy behind the actual formulae.

Define

$$E_j^i \equiv t_j (b_j - b_j^i) P^i \tag{5}$$

as province i's equalization entitlement for category j.

Notice that E_j^i might be positive, and it might be negative. It will be positive if (and only if) province *i* has a **smaller than average** tax base per capita in category *j*. In fact, it is proportional to the difference between the average tax base per capita in Canada in the category, and the tax base per capita in province *i*.

Now notice a useful property of the definition (5). Suppose that the equalization entitlements in some category are added up over all 10 provinces : the total of these equalization entitlements would be

$$\sum_{i=1}^{10} E_j^i = t_j [b_j \sum_{i=1}^{10} P^i - \sum_{i=1}^{10} b_j^i P^i]$$
(6)

Now the right side of equation (6) can be written

$$t_j [b_j P - \sum_{i=1}^{10} B_j^i]$$
(7)

But, by definition, $\sum B_j^i \equiv B_j$. And definition (3) implies that

$$b_j P = B_j$$

So expression (7) implies that

$$\sum_{i=1}^{10} E_j^i = 0 \quad i = 1, 2, \cdots, 5 \tag{8}$$

That is, this "ideal" equalization programme would be **self-financing** in each category. In each category, the negative entitlements of the provinces with high tax bases per capita exactly balance out the positive entitlements due to provinces with low tax bases per capita.

Now the derivation of equation (8) did not really use the fact that equalization entitlements are proportional to the average tax rate in Canada for that category of tax base, t_i .

But the presence of that factor of proportionality implies a nice feature for the formula. Suppose that province *i* received a grant, equal to its equalization entitlement E_j^i . Suppose as well that it chose to levy a tax rate on tax base *j* exactly equal to the national average tax rate t_j in this category. How much revenue would the province receive, from its own taxes on this tax base, and from its equalization entitlement payment? Its total revenues, given a tax rate of t_j , would be,

$$t_j B_j^i + E_j^i = t_j B_j^i + t_j [b_j - b_j^i] P^i]$$
(9)

Since $B_j^i = b_j^i P^i$, the first and last term in expression (9) cancel : the province's total revenues, from its own tax base, and from its equalization entitlement, would be

$$t_j b_j P^i = r_j P^i \tag{10}$$

Expression (10) says that receiving a grant equal to the equalization entitlement would have the following effect : the province would be able to get exactly the national average revenue per person from that revenue source, **should it choose to levy the average tax rate** on that revenue source. In other words, giving grants, in each category, equal to the province's equalization entitlement in that category, equalizes the tax bases per capita across provinces. In a sense, basing grants on equalization entitlements eliminates differences in fiscal capacity across provinces : after such a programme, any differences in expenditure (and taxation) across provinces would be based on differences in residents' tastes. With such a system, no province could claim that it was unable to provide the same public services as wealthier provinces.

Of course, there are 5 categories here. So the total grant that province i would receive, under this "ideal" equalization programme, would be

$$E^i \equiv \sum_{j=1}^5 E^i_j \tag{11}$$

the sum of its equalization entitlements in all categories. For a given province, some of these equalization entitlements E_j^i would be positive, and some negative. A province might be relatively rich in one revenue source, put relatively poor in another. In calculating the total equalization E^i to each province, the negative entitlements in some categories would cancel out some of the positive entitlements in others. E^i would be positive if (and only if) the province were a "have not" province on average, over all 5 revenue categories.

In particular, one of the 5 categories is "resource revenues", that is the income the provincial government collects from taxes and royalties on mineral, oil, timber, and other natural resources. Ontario, for example, is a "have" province in terms of personal income tax (personal income per capita is higher in Ontario than the Canadian average), but a "have–not" province in terms of resource revenue (we don't have much oil).

Since equation (8) holds for each of the 5 categories, it holds when added up over all 5. Under "ideal" equalization, the total net equalization payments, added up over all the provinces, would be zero : the negative grants to rich provinces would exactly cancel out the positive grants to poor provinces.

$$\sum_{i=1}^{10} E^i = 0 \tag{12}$$

The Actual Canadian System

The actual system of equalization used in Canada prior involves two significant modifications to the "ideal" system described above. That ideal system is **self-financing** : "have-not" provinces receive money from "have" provinces, and the payments made by the "have" provinces (those with negative E^i values) exactly pay for grants made to "have not" provinces. [This "ideal" system was **never** the system actually used in Canada.]

So one modification to the ideal system is that formulae (5) and (11) are not used for "have" provinces, only for "have–not" provinces. Any province for which $E^i > 0$ receives exactly $E^{i,2}$. But any province for which $E^i < 0$ does not have to pay anything into the system. So all positive equalization entitlements are paid, and all negative equalization entitlements are forgiven.

Unlike the ideal system described above, this actual Canadian system is not self-financing. Some provinces receive, but no provinces give. So where does the money come from? From the federal government. The federal government pays money to the "have not" provinces, money which comes from general federal government tax revenues. "Have" provinces pay nothing (and receive nothing).

Of course there is no free lunch here. The money being paid out in equalization comes from general tax revenues, which means that taxpayers in all the provinces pay for it. Although the Alberta government has to pay nothing into the plan, even though Alberta has consistently had equalization entitlements E^i which were negative, the taxpayers of Alberta pay for about 17% of the cost of equalization, since Albertans pay about 17% of federal tax revenues.

Taxpayers in the "have not" provinces also pay for equalization. But, on net, they are gainers. At the margin, each additional \$1 in equalization paid to New Brunswick would cost New Brunswick taxpayers about 3 cents, since New Brunswick accounts for about 3 percent of the Canadian economy.

By construction, the net amount of money paid into an ideal system is constant : it's zero. But the system actually used in Canada was not constructed to have a constant revenue requirement. If the gap widened between "have" and "have not" provinces, then total equalization payments would increase. In some year, suppose that E^1 , which had been positive, increased by \$1 million. The formula defining the E^i 's implies that some other provinces' E^i 's would have to decrease, because equation (12) must always hold. But what the federal government actually paid under the

 $^{^{2}}$ subject to the other major modification, to be discussed below

representative national average system were only the positive E^i 's. If the increase in the positive E^1 were offset by a decrease in the negative E^2 , then total federal government equalization payments would have to rise.

Starting in the 1970's the world price of oil increased substantially. Oil and gas royalties were, and are, a major revenue source for the Alberta government. By 1970, Alberta had long been established as a "have" province, one for which $E^i < 0$. The increase in the world oil price increased the value of all the tax bases B_j^i which were based on oil and gas revenues. ³ So suddenly, the equalization entitlement of each "have not" province became a lot larger. (And Alberta's negative entitlements became a lot more negative.) To see this, look at equation (5). For oil and gas revenues, b_j^i was close to (or actually equal to) 0 in many provinces. An increase in Alberta's oil revenues would increase the national average b_j , leave these "oil poor" provinces' b_j^i 's at 0, and therefore would increase E_j^i .

So the federal government suddenly had to pay out a lot more money in equalization to the "have not" provinces. This increase put a lot of pressure on the federal government budget : if the federal government deficit were to be kept under control, federal taxes would have to increase. Instead, the federal government adjusted the equalization formula. Rather than giving each province the E^i calculated from formulae (5) and (11), it has imposed a series of adhoc modifications, designed to cap the amount it was paying out in equalization (and to keep Ontario from receiving any equalization payments).

What is currently done is to equalize only 50 percent of resource revenues. That is, with resource revenues as category #5, then the equalization entitlements for categories 1 - 4 are defined by equation (5), but for resources, equation (5*R*) is used :

$$E_5^i \equiv (0.5)t_5(b_5 - b_5^i)P^i \tag{5R}$$

The difference between (5) and (5R) is the factor (0.5) : 100% of income tax and consumption tax (and other taxes in categories 1 - 4) revenues are equalized, but only 50% of resource revenues. This special treatment of resource revenues is done to reduce the total amount of equalization which the federal government must pay. If (5) were used instead of (5B), then Ottawa would owe a lot more money to Québec and New Brunswick and Manitoba, for example. Using (5) instead of (5B) would also make Alberta's negative equalization entitlement even bigger. But that effect on Alberta doesn't matter : "have" provinces do not have to pay directly into the programme, and they never have had to.

There are (of course) many more details in the actual Canadian equalization formula. There is a rule which says that no province can collect equalization, if it would have been a "have" when 100% of resource revenue was equalized. That is, if some province had a negative total entitlement E^i when formula (5) was used for all 5 categories, including resource revenues, then it doesn't

³ In the 1970's there were actually more categories for equalization than the current number, 5. So there actually were several different types of resource revenue which were each being equalized.

get any equalization payments, even if E^i were positive when (5) is replaced by (5R) for resource revenues.

That extra rule does not matter for Alberta ; it's already a "have" even when only 50% of resource revenue is equalized. In won't matter for Ontario : including 100% of resource revenue in the formula would may us look poorer, not richer. It does matter for Newfoundland, which has always been one of the most "have–not" provinces (in per capita terms). Newfoundland still is very poor : it certainly has very big positive entitlements E_j^i in categories 1 - 4. But it also is earning a lot of resource revenue from offshore oil, so that including 100% of resource revenue makes it look richer than Ontario.

But there is yet another detail. If a province is a "have–not" province overall, but a "have" province for resources, then resources don't count for that province. In other words, Newfoundland gets to leave out category #5 (resource revenues) in calculating its E^i : that gives it bigger entitlements if $E^i > 0$ and if $E_5^i < 0$.

Finally, there is an overall cap on how rapidly overall equalization payments can grow. Total equalization payments to all receiving provinces cannot grow more quickly than Canadian GDP. (Actually, it's the weighted average of GDP in 3 earlier years : so overall equalization payments cannot grow at a higher rate between 2013 and 2014 than GDP did in going from an average of 2009–10–11 to the average of 2010–11–12.) If this constraint is binding, all "have–not" provinces' equalization payments are scaled down proportionately.

The result is that, in 2013–14, 6 of the ten Canadian provinces will receive equalization. The table below indicates what they will receive, in total, and in per capita terms

province	total eq (million\$)	eq. per capita
PEI	340	2320
\mathbf{NS}	1458	1536
NB	1513	2000
QU	7833	966
ON	3169	232
MAN	1792	1403

So the details remain fairly complicated. But not nearly as complicated as they have been. The following (optional) section outlines some of the recent history of equalization rules in Canada.