Preference Revelation : (a) A Project of Fixed Size

When there is a public good, efficiency requires that the sum of people's marginal rates of substitution of the public good for the private good (their willingness to pay for the public good) must equal the marginal rate of transformation (the cost of the public good). If some agency — government, private firm, non–profit agency — is to try and achieve this efficient allocation, it must know people's demand curves for public goods. As Samuelson emphasized, the really big problem with public goods is not that the efficiency condition is different from the efficiency condition with private goods, but that efficiency can't be achieved without a lot of information about people's preferences.

So what actually happens in the provision of public goods such as information, national defense and broadcasting? Sometimes some of these goods (some of the ones which are excludable to some degree) are privately provided. The allocation here will not be efficient, since typically these firms exclude some people from some benefits, because the firms are not able to charge different prices to different people. Sometimes these goods are financed voluntarily — which means that they will be inefficiently under–provided if people behave in a self–interested manner in deciding their contributions. Sometimes they are provided by government. Since governments (at least those in democratic countries) are elected, the government officials have some incentive to please people if they want to get re–elected. But elections do not enable people to convey exactly their preferences over public goods to politicians.

Public good providers could try and find out exactly what are people's preferences for public goods by asking them. This seldom happens in practice. If people were asked their preferences for public goods, it seems reasonable to conclude that many people would behave in a self–interested manner in responding. That is, people would not tell the truth about their demand curves for public goods unless it was in their own interest to do so.

In other words, if people were surveyed about their demand curves for public goods, we should not assume that people would respond truthfully. Rational, selfish people should try and manipulate the survey in their own interests. That makes the survey a **game**, that is a situation in people behave **strategically**. Rational selfish people, responding to a survey, will understand the rules of the game, that is how their responses to the survey will affect the taxes they pay and the quantities of public goods that they consume. Then they will choose their answers to the survey so as to achieve the outcome most favourable to them.

"Simple" questionnaires, in which people simply are asked what are their demands for the public good, would not induce people to tell the truth (if people are clever and behave strategically). If people figured out that the government would implement some sort of benefit tax system, in which people who expressed a strong demand for the public good would pay more in taxes, then people would have an incentive to **understate** their preferences.

Suppose instead people figured out that public goods would not be finance by benefit taxation, but would be financed out of general tax revenue. Then they might not understate their own

preference. If a person learned that parks were to be financed by a cigarette tax, then if the person were a non–smoker she would realize that she would be getting the parks at no cost to herself. Assuming that cigarette taxes are born entirely by smokers, the added cost of any expansion of the park system, if financed by a cigarette tax, would be paid only by smokers. In this case, the non–smoker does not have an incentive to understate her benefits from parks. The value of the benefits which she reports in the survey don't affect her tax payments. But she would now have an incentive to **overstate** her preferences. If someone else is paying for the parks, then she might as well exaggerate the benefits she gets. That would make the government agency more likely to go ahead with the project, or to expand the project. In general, for a person to figure out what would be the best response to a survey about her demand for the public good might be pretty complicated, and it might depend on her guesses about what other people were responding. But figuring out the best strategy, although complicated, would be a better option than simply telling the truth.

However, it turns out that if the taxes people pay are based on the responses they give, and if the taxes are designed cleverly enough, then people will want to tell the truth, even if they have figured out how the system works. That is, there are mechanisms which will induce people to tell the truth about their tastes for public goods, even if they are self–interested, and even if no–one else knows their preferences — if people correctly figure out how the tax system works.

One feature that is needed for these **mechanisms** to work is that the government (or whoever is asking the questions) can **commit** to the mechanism. That is, a mechanism is a set of rules which the government announces. It will be assumed here that the government will actually use the rules it announces, and that people believe that the government will use these rules. In the example below, the government proposes rules, for how it will tax people, based on responses people give to a questionnaire. In order for this mechanism to work, the government must actually go ahead and use those rules, once the questionnaire has been answered. So it will be taken for granted here that the government will not try and change the rules once it has some information, and that people believe that the rules actually will be used.

An Example : A Facility of Fixed Size

As an example, consider the issue of whether of not to build some public facility of a fixed scale, such as a stadium or theatre. The simplification here is that there is no possibility of varying the scale of the facility : the question is an "all or nothing" issue of whether to build the facility or not. Suppose that the facility costs $100,000,000 to build. Suppose as well that the government agency is trying to make its decision (whether or not to build the facility) efficiently : to build the facility if the total dollar value of all the people's expressed benefits for the stadium exceed the cost.

If people were telling the truth, then that would be a good criterion for the government to use : the project is worth undertaking if the benefits (in dollars) from the project, added up over all

people, exceeded the cost of the project.

Assume that there are 1 million people in the city, so that the government is going to ask each person to state, in dollars, what the stadium is worth to him or her. If the sum of these announced valuations exceed \$100,000,000, then the facility will be built.

Further, assume that the cost of the stadium, if it is built, will be split equally, \$100 per person, among the city's residents.

Now if the government simply announces that the stadium will be built if the sum of the announced valuations exceeds \$100,000,000, and also announces that the cost will be shared equally among all people, people do not have an obvious incentive to tell the truth. [What are the incentives? That's left to the reader. So think about how you might want to answer a survey, if you knew that you truly valued the project at \$25, and if you knew that you would be assessed a tax of \$100 if the project were built. Now think about how you'd want to answer the survey if you knew that the project was really worth \$200 to you, and you still expected to be taxed \$100 if the project were built, regardless of your answer.]

But in the mechanism I will present here, the government puts in an **additional** tax, a tax which applies only if a person's answer affects the decision. That tax is assessed in addition to the share of the project which a person will pay, if the project is built. And this additional tax might be assessed even if the project is not built.

Some notation : for person 1, let $\tilde{V}_1$ denote the total of *everyone else's* **announced** valuation for the facility, leaving out person 1's own announced valuation. Let v_1 be the valuation that person 1 announces. So

$$\tilde{V}_1 \equiv v_2 + v_3 + \cdots + v_{1000000}$$

where v_i is what person i says that the project is worth to her. (Of course what she says it's worth to her may not be what she really thinks that it is worth to her : we don't know her true preferences, just what she tells us her preferences are.)

One of the rules the government will stick to is that the facility will be built if the some of people's announced values exceeds the cost of the facility, which is \$100,000,000. From the definition above of $\tilde{V}_1$, as the sum of everyone else's announced value (except for person 1's), that rule can be written : build the facility if and only if

$$\tilde{V}_1 + v_1 \geq 100,000,000$$

If the facility is built, then everyone will pay a tax of \$100 to pay for it. But further, person 1 may pay a special tax, if her response turns out to be **pivotal**, that is if her response affects the decision whether or not to build the facility.

Specifically, if

$$\tilde{V}_1 < 99,999,900$$

and if

$$\tilde{V}_1 + v_1 > 100,000,000$$

then person 1's response is **pivotal** : the facility would not have been built without her response being as big as it was. In other words, if the announced values of all the other people were less than \$100 per person, then the facility won't be built, since the cost per person is greater than the benefit per person (or at least it's less than the average value of what people say are their benefits). In these circumstances, if person 1 says that her benefit is really high, then, once her announced benefit is taken into account, the average benefit of all 1,000,000 people will be greater than the cost per person, so the facility will be built.

Of course, this may not happen. But if it does happen that $\tilde{V}_1 < 99,999,900$, and that $\tilde{V}_1 + v_1 \geq 100,000,000$, then person 1 is said to be pivotal, since her response to the survey affects the overall decision.

In that case, she will have to pay a special "pivot tax" of

$$99,999,900 - \tilde{V}_1$$

on top of the regular tax of \$100. As well, if

$$\tilde{V}_1 \geq 99,999,900$$

and

$$\tilde{V}_1 + v_1 < 100,000,000$$

then person 1 would again be pivotal, since in this case her low announced valuation keeps the facility from being built. In this case, she would also have to pay a pivot tax, this time equal to

$$\tilde{V}_1 - 99,999,900$$

(In this case, she is paying a tax, even though the facility is not being built, just because she influenced the decision not to build the facility by announcing such a low value).

Why \$99,999,900, and not \$100,000,000 in calculating the pivot tax? Because we're going to build the facility only if people's **average** valuation exceeds the cost per person of \$100. I'm pivotal if the average valuation, not including mine, is less than \$100, and if my announced valuation pulls the average above \$100 — or if the average valuation, not including mine, were greater than \$100, and if my announced valuation pulls the average below \$100.

Now, it might seem that this extra tax would not induce a person to want to tell the truth. It might seem that it would induce her to want to understate her preferences, to avoid the "pivot" tax.

But suppose that she does understate her true valuation. Suppose, for example, that the facility really is worth \$200 to her. Would it pay her to lie, say to state a valuation of only \$100?

The effect of her lie depends on what everyone else has stated. If $\tilde{V}_1$ is greater than \$100,000,000, then the facility gets built whatever she says is her valuation. In this case, she is **not** pivotal, the facility gets built no matter what she says, and she pays \$100 as her share of the

cost. Since she won't be stuck with the pivot tax, and since the facility will be built whether she tells the truth or lies, then there is no incentive to lie. Remember : a person pays a pivot tax only if her answer changes the result.

What if $\tilde{V}_1 < 99,999,800$? Then if she tells the truth ($v_1 = 200$), the facility will not get built. She can get it built if she exaggerates her valuation enough — but then she'll get hit with a pivot tax. For example, if

$$V_1 = 99,999,700$$

and she announces $v_1 = 400$, her lie gets the facility built, since it pushes the sum of announced valuations above \$100,000,000. That gets her a facility, a facility which is worth \$200 to her — but she would have to pay taxes of \$300, the regular \$100 tax plus a pivot tax of $\$99,999,900 - \$99,999,700 = \$200$. So it's not in her interest to overstate her benefit. Overstating only makes a difference if she is pivotal. And if she is pivotal, then the extra pivot tax she pays for affecting the decision is greater than the benefit she gets from the facility.

If $\tilde{V}_1$ is between \$99,999,800 and \$99,999,900, then person 1 would be pivotal if she tells the truth (remember: her true valuation here is assumed to be \$200). She can avoid the pivot tax by understating her valuation. For instance, if $\tilde{V}_1 = 99,999,850$, she could announce a valuation of \$100, which would mean that the facility would not be built, and she would avoid paying any pivot tax. But in this instance, her pivot tax is only \$50 ($99,999,900 - \tilde{V}_1$). She would be better off telling the truth, even though it means paying her regular tax of \$100 **and** her pivot tax of \$50, because then she would get to have a facility which is worth \$200 to her : telling the truth, and paying a total tax of \$150 to get a facility built which is worth \$200 is a better strategy than understating her preferences, paying nothing, and getting nothing..

Finally, if $\tilde{V}_1$ is between \$99,999,900 and \$100,000,000, she could prevent the facility from being built by understating her true valuation enough. But that would be bad for her in two ways. First of all, she won't get the facility. Secondly, in this case her lie would make her pivotal, since her lie, announcing a low valuation was the reason that the facility was not built. So telling the truth gets her a facility worth \$200, at a cost of \$100, in this case. Understating a lot gets her no facility, and a pivot tax as well!

So what should she do?

The odds are that she won't have any influence on the outcome. In that case, it doesn't matter whether she lies or tells the truth. She is only 1 person among 1,000,000. Her answer only matters if the other valuations average out close to \$100 per person, so that she could be pivotal. And in that case, the analysis above shows that she is better off telling the truth than lying. The pivot tax she might end up paying can never exceed \$100 if she tells the truth (if her true valuation is \$200), so she would be better off telling the truth, affecting the result, getting the facility built, and paying the pivot tax, than understating her preference to dodge a small pivot tax.

Notice that the pivot tax does not depend specifically on what she states as her valuation, only on whether she affects the result, and on how that change affects other people. For example,

if

$$\tilde{V}_1 = 99,999,850$$

then her pivot tax will be

$$\tilde{V}_1 - 99,999,900 = 50$$

whenever she is pivotal in getting the facility built, that is whenever $v_1 > 150$. Understating her valuation slightly (say stating it's \$180 instead of her true \$200) won't reduce her pivot tax : it's \$50 whenever $v_1 > 150$. The only way understating reduces her pivot tax is if she states $v_1 < 150$, in which case she dodges the pivot tax — but also doesn't get her facility.

Now person 1 actually doesn't know what everyone else has stated, at the time she is asked her valuation. She has to decide her announcement, v_1, without knowing what is her $\tilde{V}_1$ (the sum of everyone else's v_i's). But the paragraphs above show that she really doesn't need to know what other people have announced. Either lying about her valuation will have no effect at all, on either whether the facility gets built, or on her taxes (the most likely situation), or it will have an effect. And if it will have an effect, she is strictly better off telling the truth than lying. In other words, given the rules of this tax scheme, telling the truth is a **dominant strategy** for person 1. Whatever other people's valuations are, and regardless of whether they tell the truth or lie, there is no strategy for person 1 which is any better than simply telling the truth about what the facility is worth to her.

Person 1's decision process is no different than any other person's. Person 2 has a similar situation : he will have to pay a pivot tax if the average of the other 999,999 people's announced valuations is less than \$100, and his announcement pushes the average over \$100, or if the other people's average valuation is more than \$100, and his low announced valuation pushes the overall average below \$100. So each person will, if she or he understands the tax rules correctly, want to tell the truth. Even though the government has no idea about any person's true valuation, it can design a tax scheme which — if understood correctly — will induce everyone to reveal their preferences, because it is in each person's self–interest to do so.

So who's pivotal? That depends on the distribution of people's actual valuations. It might be nobody. For example if we had 100 people who valued a facility at \$102, 100 people who valued it at \$99, and if the facility cost \$20,000 to build, then no individual will pay a pivot tax if everyone tells the truth.. (You should check that, in this case, $\tilde{V}_i + v_i > 19,900$ for each person, if everyone tells the truth.) It could be that a lot of people are pivotal : if 10 people valued a facility at \$151, 10 valued it at \$50, and if the facility cost \$2000, then each high–valuation person would face a $\tilde{V}_i$ of \$1859¡\$1900, and would pay a pivot tax of \$41 if everyone told the truth.

What does the government do with the pivot tax revenue? Notice that this pivot tax revenue (if there is any) would be money which is over and above the cost of the facility. That may seem like a nice situation, but the government actually would have to be careful not to return this excess to the taxpayers. Why? If taxpayers realized that they would be getting a share of the pivot tax revenue, then they might start to adjust their responses, so as to make other people pay more

pivot taxes. Figuring this effect out would be very complicated, but if people were really clever it would (very slightly) offset the incentive to tell the truth.