Social Insurance and Pensions : (a) The Basic Model of Insurance

In examining the demand for insurance, economists look at the choices made by **risk averse** people, who maximize **expected utility**. That is, we assume that people's behaviour obeys the **expected utility hypothesis** (due to von Neumann and Morgenstern).

This expected utility hypothesis describes the choices people might make under uncertain circumstances. Let π be the probability of some uncertain event, such as a person's house burning down, or the person losing her job. Then $1 - \pi$ is the probability that the event does not occur. (So if $\pi = 0.10$, then there is a 10 percent chance that the person will lose her job, and a 90 percent chance she will keep her job.) Let y_G denote the person's income in the "good" state of the world, and y_B her income in the "bad" state of the world. So if her job paid $50,000 a year, and if she had no form of employment insurance, or other income, then she would have $y_G = 50000$ and $y_B = 0$, if the good state were the state of the world in which she had a job, and the bad state of the world were the state in which she lost her job.

If a person buys insurance, she will lower y_G, since the person has to pay premiums for the insurance, and she will increase y_B, since the person will collect money if her house burns down (or if she loses her job). So if an insurance policy costs $100, and the policy pays the person $2000 if her house burns down, then buying the insurance policy would lower y_G by 100 (the premium she has to pay), and would increase y_B by 1900 (the amount she collects in insurance, minus the cost of the insurance). So the choice of whether to buy insurance, or how much insurance to buy, is really a choice of which (y_G, y_B) combination to have. Suppose, for example, that a person could actually choose to buy private employment insurance from some insurance company, paying a premium for a policy which would compensate her if she were laid off from her job. If it cost her $5000 to buy this insurance, and if the policy promised to pay her $25,000 in the event of her being laid off [so that the premium was $5000, and the value of the policy was $25,000], then if she bought the insurance it would mean that she prefers the combination $y_G = 45000$, $y_B = 20000$ to the combination $y_G = 50000$, $y_B = 0$ that she would have if she did not choose to buy the insurance.

The expected utility hypothesis is the hypothesis that there is some **utility–of–income** function $u(y)$, with $u'(y) > 0$, such that the person will always choose the (y_G, y_B) combination which maximizes her expected utility

$$EU \equiv (1 - \pi)u(y_G) + \pi u(y_B)$$

In other words, there is some function $u(\cdot)$ which translates this person's income into "happiness". And the person chooses actions so as to make the expected value of this"happiness index" as high as possible. [von Neumann and Morgenstern proved mathematically that : if a person's behaviour satisfied some "reasonable" axioms, then she must behave as an expected utility maximizer, for **some** utility–of–income function $u(\cdot)$. The exact shape of her utility–of–income function would depend on her attitude towards risk.]

EU is the person's **expected utility**, and it will depend on the income combination (y_G, y_B), as well as on the probability π that the bad event will happen. As well, of course, it depends on the shape of her utility–of–income function $u(\cdot)$. An expected utility maximizer is said to be risk averse if (and only if) her utility–of–income function $u(y)$ is **concave** : $u''(y) < 0$. (If $u''(y) > 0$ then she is a **risk lover**, and if $u''(y) = 0$ then she is **risk–neutral**.)

Graphically, if a person is an expected utility maximizer, then we could draw indifference curves in (y_G, y_B) space, indicating all the (y_G, y_B) combinations which give her the same level of expected utility. For example, the combinations which give her an expected utility level of 100 are all the (y_G, y_B) combinations such that

$$(1 - \pi)u(y_G) + \pi u(y_B) = 100 \tag{1}$$

Equation (1) defines a downward–sloping curve, with a slope of

$$\frac{dy_B}{dy_G}|_{EU=100} = -\frac{1-\pi}{\pi}\frac{u'(y_G)}{u'(y_B)} \tag{2}$$

if y_G is graphed on the horizontal and y_B on the vertical. Equation (2) shows that the indifference curves will have the usual convexity — getting steeper as y_B increases and y_G decreases — if $u''(y) < 0$.

A person's expected utility is **not** the same thing as her **expected income**. Her expected income from a combination (y_G, y_B) is defined as

$$Ey \equiv (1 - \pi)y_G + \pi(y_B)$$

The expected income is, in a sense, the person's "average income" : if there's a 10 percent chance she loses her job, and a 90 percent chance she keeps it, then the "average" value of the combination $y_G = 50000$, $y_B = 0$ is \$45000.

An insurance policy is said to be **actuarially fair** if it does not change a person's expected income. If the probability of a layoff is 10%, then the policy in the example above, in which premiums were \$5000, and in which she got net benefits of \$20000 while unemployed, is **not** actuarially fair : it reduces her expected income from \$45000 to $(0.9)(45000)+(0.1)(20000) = 42500$. If an insurance policy is actuarially fair, then it means that the insurance company would make zero profit, on average, on the policy. In the example, if many identical people purchased this policy, then on average the company would be collecting \$5000 each from 90 percent of them, and paying \$20000 each to 10 percent of them, which means it would be making profits of \$2500 per person.

The math also shows that if the insurance company makes zero expected profits on a policy, then it leaves unchanged the person's expected income. If a policy paid Y if a person were unemployed, and cost a premium of X when she was employed, then the expected profit from the policy is

$$profit = (1 - \pi)X - \pi Y$$

The person's expected income if she took such a policy would be $y_G = 50000 - X$, and $y_B = Y$, so that

$$Ey = (1-\pi)(50000 - X) + \pi Y = (1-\pi)50000 + [\pi Y - (1-\pi)X] = 45000 - profit$$

The set of all income combinations from policies which are actuarially fair, that is the set of all policies which make zero expected profit, lies on a line with slope $-(1-\pi)/\pi$ — when y_G is graphed on the horizontal, and y_B on the vertical. In this case, for example, a policy is actually fair if it gives the person an expected income of 45000, that is if it gives her a combination (y_G, y_B) with

$$(1-\pi)y_G + \pi y_B = 45000 \tag{3}$$

since her expected income (without any insurance) is \$45000. Equation (3) is the equation of a line with slope $-9 = -(1-0.1)/0.1$ through the no–insurance combination $(50000, 0)$.

If a person can buy any amount of insurance, as little or as much as she wants, at an actuarially fair price, then the set of income combinations that she can get is represented by this line, with slope $-(1-\pi)/\pi$. Given such a set of options, she would choose the policy which gets her to the highest indifference curve.

The 45–degree line, in the (y_G, y_B) diagram, represents income combinations such that the person is **fully insured**. If she is fully insured, then her income is the same whether or not she is laid off, so that $y_G = y_B$. Equation (2) shows that the slope of her indifference curves must be $-(1-\pi)/\pi$ along the 45–degree line (and only there : if $u'' < 0$, and if $y_G \neq y_B$, then $u'(y_G)/u'(y_B) \neq 1$).

Therefore,

Result : Any risk–averse expected utility maximizer will purchase full insurance, if insurance is available at an actuarially fair price.

If insurance is available, but only at a price which is higher than the actuarially fair price, then a risk–averse expected utility maximizer would want to purchase less than full insurance (or maybe no insurance at all) : her indifference curve will still have a slope of $-(1-\pi)/\pi$ along the 45–degree line, but the budget line has a slope which is less than $-(1-\pi)/\pi$.

Figure 1 illustrates the case of a person buying some insurance, but not complete insurance, when the price is higher than the actuarially fair price.

If there is a competitive insurance industry, and if insurance companies and their customers all know (and agree on) the probability π of the bad event, then the competition will drive insurance companies' economic profits to zero. (If a company were selling a policy that made a profit, then some other firm would have an incentive to offer the same policy at a slightly lower price, still making a profit, but stealing away all the first company's business. In this way, competitors would undercut each other until all profits are bid away.) A policy makes zero profits if it offers customers the same expected income as they would get in the absence of insurance. So competition among insurance companies (and perfect information) imply the buyers of insurance get to choose from policies yielding income combinations on a line with slope $-(1-\pi)/\pi$ through the "endowment point", the point where they would be if they purchased no insurance.

Given the actuarially fair odds, customers would all want to purchase full insurance. Even if they were able to buy less than full insurance, or more than full insurance, they would not choose to do so, since the highest indifference curve they can achieve on their budget set is at the tangency along the 45–degree line.

Figure 2 illustrates : as long as the person is risk averse, her indifference curve will be tangent to the budget line along the 45–degree line, which is the set of outcomes at which her income is the same in both states of the world.

If competition among insurance companies did not drive the price of insurance down to zero, then the budget line buyers would face would be less steep than $-(1-\pi)/\pi$. In such a case, they would choose to buy incomplete insurance. A policy with a deductible amount is an example of purchase of incomplete insurance.