Public Goods : (b) Efficient Provision of Public Goods

Efficiency and Private Goods

Suppose that there are only two goods consumed in an economy, and that they are both **pure private goods**. Suppose as well that there are only two people in the economy. So this is the economy analyzed in intermediate microeconomics (AP/ECON 2300 and 2350), often using the **Edgeworth box diagram** if the supply of the two goods is fixed.[1]

However, here the supply of the two goods is **not** fixed. Instead, there is some production technology in the economy, and some endowment of inputs to production (such as labour and machinery), which determines which combinations of the two goods can be produced. To represent the production possibilities, the **production possibility frontier**, sometimes called the "production possibility curve" is used, again as in intermediate microeconomics. [1] If X and Y are the total quantities produced of the two goods, the production possibility curve shows the (X, Y) combinations which are feasible in the economy. This curve could be represented by some equation

$$X = F(Y)$$

showing the maximum quantity X of food which could be produced, given that we are producing Y units of clothing in the economy. The production possibility frontier slopes down : the more clothing that is produced, the less food which can be produced from the economy's limited resources. So

$$F'(Y) < 0$$

$-F'(Y)$ also represents the opportunity cost of clothing, in terms of foregone food. Increasing the economy's clothing production by Δ units would require the economy to produce $-F'(Y)\Delta$ fewer units of food. This opportunity cost is usually referred to as the **marginal rate of transformation**, or the MRT, since it is the rate at which food can be transformed into clothing.

$$MRT \equiv -F'(Y)$$

In this simple two–good, two–person economy, an allocation consists of 4 numbers, representing how much of each good each person gets : x_1, x_2, y_1 and y_2. (So that, for example, y_1 is the quantity of clothing consumed by person # 1.) If X and Y are the total quantities produced of the two pure private goods, then

$$x_1 + x_2 \leq X$$

$$y_1 + y_2 \leq Y$$

[1] See chapter 31 in (the 8th edition of) Varian's intermediate micro text, for example.

[1] See chapter 32 in Varian's intermediate micro textbook (8th edition), for example.

since the two goods are rival.

As long as food and clothing are goods, and not bads, we certainly want to have people consume all the food and clothing that is produced. So the inequalities above can be replaced by the equations

$$x_1 + x_2 = X \tag{priv1}$$

$$y_1 + y_2 = Y \tag{priv2}$$

Given the production possibility frontier, an allocation (x_1, x_2, y_1, y_2) is feasible only if

$$X \leq F(Y)$$

Again, there is no point in using scarce resources to produce goods unless the goods are to be consumed, so that the inequality above can be treated as an equality. Using equations ($priv1$) and ($priv2$), this resource constraint becomes

$$x_1 + x_2 = F(y_1 + y_2) \tag{priv3}$$

An efficient allocation might maximize some **social welfare function** [3], depending on the utility functions of the two people. That is, person 1's preferences are represented by some function $U^1(x_1, y_1)$ of her consumption of food and clothing, increasing in the quantities consumed of both goods. Similarly person 2's preferences could be represented by some utility function $U^2(x_2, y_2)$ defined over his consumption of food and clothing. A benevolent social planner might want an allocation which maximizes the social welfare function

$$W(U^1, U^2)$$

depending on the well–being U^1 and U^2 of the two people.

But the allocation must be feasible ; that is, it must satisfy constraint ($priv3$). So an efficient allocation (x_1, y_1, x_2, y_2) is one which maximizes

$$W(U^1(x_1, y_1), U^2(x_2, y_2))$$

subject to the feasibility constraint ($priv3$).

To solve such a constrained maximization mathematically, set up the Lagrangian

$$\mathcal{L}(x_1, y_1, x_2, y_2; \lambda) = W(U^1(x_1, y_1), U^2(x_2, y_2)) + \lambda(F(y_1 + y_2) - x_1 - x_2)$$

and maximize it with respect to x_1, y_1, x_2, y_2 and the Lagrange multiplier λ. Maximization this Lagrangian with respect to x_1 (and each of the other variables) means setting the derivative of $\mathcal{L}$

[3] as in chapter 33 of the eighth edition of Varian's intermediate microeconomics text

with respect to that variable equal to zero. So the first–order conditions for this maximization are :

$$W_1 U_x^1 = \lambda \tag{$x1$}$$

$$W_1 U_y^1 = -\lambda F'(y_1 + y_2) \tag{$y1$}$$

$$W_2 U_x^2 = \lambda \tag{$x2$}$$

$$W_2 U_y^2 = -\lambda F'(y_1 + y_2) \tag{$y2$}$$

where W_1 and W_2 are the derivatives of the social welfare function with respect to the utility of people 1 and 2, U_x^1 is person 1's marginal utility of food consumption, and so on.

Now recall that a person's **marginal rate of substitution** is (the negative of) the slope of her indifference curve :

$$MRS^1 \equiv \frac{U_y^1}{U_x^1}$$

$$MRS^2 \equiv \frac{U_y^2}{U_x^2}$$

Now if we divide the left side of equation ($y1$) by the left side of equation ($x1$), we get the efficiency condition

$$MRS^1 = -F'(y_1 + y_2) = -F'(Y) = MRT \tag{$priv4$}$$

and if we divide the left side of equation ($y2$) by the left side of equation ($x2$), we get

$$MRS^2 = -F'(y_1 + y_2) = -F'(Y) = MRT \tag{$priv5$}$$

Equations ($priv4$) and ($priv5$) are the efficiency conditions for allocations in an economy when all the goods are pure private goods. There are really two conditions contained in them : (i) each person's marginal rate of substitution between any two goods should be the same as any other person's marginal rate of substitution between those two goods, and (ii) each person's marginal rate of substitution between any two goods should equal the marginal rate of transformation between the goods. These conditions characterize efficiency even if there were many more than two goods, and many more than two people — as long as all the goods were rivalrous.

One way of interpreting the efficiency conditions is to treat one good, say food, as **numéraire**, that is, to measure everything in terms of food. That means that person 1's marginal rate of substitution MRS^1 is the price she is willing to pay (relative to the price of food) for a little more clothing. The marginal rate of transformation is cost of producing a little more clothing (in units of food). The efficiency conditions say that every person should, at the margin, want to pay the same price for a little more clothing, and that that price should equal the cost of producing a little more clothing.

That's all old hat. Now what about public goods?

Efficiency and a Public Good

As above, the economy still is a two–person, two–good world. But instead of two pure private goods X and Y, now we have one pure private good X, and one **pure public good** Z.

There is no difference on the production side. Whether we are producing television programmes or clothing, we must use scarce resources such as people's labour, and machinery. So the production side can again be represented by a production possibility curve, with an equation

$$X = F(Z)$$

showing the maximum quantity of food that can be produced, given the quantity of television programming that is being produced. This production possibility curve still has a negative slope, $F'(Z) < 0$, and $-F'(Z)$ still represents the opportunity cost of television programming, in terms of foregone food. Again, this opportunity cost will be described as the **marginal rate of transformation** of food into television programming.

$$MRT \equiv -F'(Z)$$

As before, an allocation is simply the quantities each of the two people gets of each of the two goods : x_1, x_2, z_1 and z_2.

As before, we can only allocate quantities of the private good that we have produced

$$x_1 + x_2 \leq X$$

Since food is assumed to be a good, and not a bad, this restriction can again be treated as an equality :

$$x_1 + x_2 = X \qquad (pub1)$$

But now good Z is non–rival. The restriction on what the two people can consume becomes

$$z_1 \leq Z \quad ; \quad z_2 \leq Z$$

Why? How much television programming can person #1 watch? She can only watch programmes that have been produced. Since Z is the number of hours of programming which have been produced, it certainly must be the case that $z_1 \leq Z$.

But that is the only restriction on how much television she can watch. How much she can watch does not depend at all on how much person #2 watches. Person #2 can increase his television watching without affecting the opportunities person #1 has for television watching.

If the public good Z really is a "good", that means that people's utilities are increasing in z_1 and z_2. We want to allocate them as much as possible of the goods, so that each person should consume the full amount available of the public good. So the feasibility condition on the consumption of television programming could be written

$$z_1 = z_2 = Z \qquad (pub2)$$

Now the social planner gets to pick x_1, x_2, z_1 and z_2 to maximize the social welfare function. As before, social welfare depends on the well–being of each person, and each person's well–being depends on her consumption of the goods. So, as before, the social planner's welfare function can be written $W(U^1, U^2)$, where U^1 and U^2 are the utility levels of the two people, and the utility levels can be written $U^1(x_1, z_1)$ and $U^2(x_2, z_2)$.

The social planner is still constrained by the production possibility frontier. Using equation ($pub1$), this constraint could be written

$$x_1 + x_2 = F(Z) \tag{$pub3$}$$

(where again we want an equality, since there is no point in using scarce resources to produce goods that will not be consumed).

So, when one of the goods is a pure public good, an efficient allocation is one which maximizes some social welfare function $W(U^1(x_1, z_1), U^2(x_2, z_2))$, subject to the feasibility constraint ($pub3$). But the condition ($pub2$) on the consumption of television programming means that it will always be efficient to let each person consume the maximum available amount of television programming. Given that $z_1 = z_2 = Z$, an efficient allocation can be found by maximizing $W(U^1(x_1, Z), U^2(x_2, Z))$, subject to the production constraint ($pub3$). To solve this, again the Lagrangian should be set up

$$\mathcal{L}(x_1, x_2, Z; \lambda) = W(U^1(x_1, Z), U^2(x_2, Z)) + \lambda(F(Z) - x_1 - x_2)$$

and it should be maximized with respect to x_1,x_2, Z and λ. So, taking partial derivatives of the Lagrangian, and setting them equal to zero, the first–order conditions for efficiency are

$$W_1 U_x^1 = \lambda \tag{$x1$}$$

$$W_2 U_x^2 = \lambda \tag{$x2$}$$

$$W_1 U_z^1 + W_2 U_z^2 = -\lambda F'(Z) \tag{Z}$$

As before, we can define a person's **marginal rate of substitution** between the goods as the negative of the slope of her indifference curve :

$$MRS^1 \equiv \frac{U_z^1}{U_x^1}$$

$$MRS^2 \equiv \frac{U_z^2}{U_x^2}$$

MRS^1 is the price — in units of food — that person 1 is willing to pay for a little more of the public good. (How much she is willing to pay will depend on how much of the public good she already has, and on how much of the private good she has.) Equations ($x1$) and ($x2$) above imply that

$$MRS^1 = W_1 \frac{U_Z^1}{\lambda}$$

$$MRS^2 = W_2 \frac{U_z^2}{\lambda}$$

if the allocation is efficient. Now divide both sides of equation (Z) by λ to get

$$W_1 \frac{U_z^1}{\lambda} + W_2 \frac{U_z^2}{\lambda} = -F'(Z)$$

But, from the equations immediately above, this optimality condition can be written

$$MRS^1 + MRS^2 = MRT \qquad (pub4)$$

Equation $(pub4)$ is often described as the **Samuelson condition** for efficiency in an economy with pure public goods. It says that, if the allocation is efficient, then the **sum** of all people's marginal rates of substitution between a pure public good and a pure private good should equal the marginal rate of transformation between the goods.

This condition is **not** the same as the efficiency conditions $(priv4)$ and $(priv5)$ for an economy with only private goods.

Wiith this new efficiency condition, a person's marginal rate of substitution between a pure public good and a pure private good does not necessarily equal each other person's MRS between the two goods. And the cost of a marginal increase in the quantity produced of the pure public good should not equal any person's marginal willingness to pay for a little more of the public good : it should equal the sum of **everybody's** marginal willingness to pay for a little more of the public good.

What's Changed because of the Pure Public Good?

In going from a world with two pure private goods, to a world with a private good and a public good, it seems that the optimality conditions have "flipped". In the two–private–good case, the **quantities** people consume of the private good Y should be added together to check on feasibility ; all the people's *MRS*'s should be equal for efficiency. In the world with a pure public good, all the people's *MRS*'s should be added together for efficiency ; the **quantities** people consume of the public good are set equal (to the quantity supplied) to check on feasibility.

What would happen when we have, simultaneously, more than one pure private good, and one or more pure public goods in the economy? If the allocation is efficient, then equations $(priv4)$ and $(priv5)$ should hold for each pure private good, and equation $(pub4)$, the Samuelson condition, should hold for each pure public good.

Why the difference? Consider the costs and benefits of producing a little more of some good. Given the scarcity of resources in the economy, the costof producing a little more is the opportunity cost, the MRT. If we produce a little more of the good, we will have to produce a little less of some other good. So whether the good for which we are expanding production is private or public, the cost of the production expansion is MRT per added unit. With a pure private good, the benefit of one more shirt, or one more haircut, will go to one person only, the person who consumes the added unit. Efficiency requires that the marginal benefit equal the marginal cost. (If not, then we should expand production of the good more if the MRS exceeds the MRT, and contract production if the MRS is less than the MRT.) Efficiency also requires that each person's MRS be the same : if one person valued shirts more highly than another (compared to the numéraire good, food), then both people could be made better off if they traded shirts for food with each other.

On the other hand, if we produce a little more of a pure public good, then the benefits do not go to just one person. The non–rivalrous nature of the good means that every person will be able to consume a little more. So the benefit to society of producing a little more of the public good is the sum of the marginal benefits of all the people. In general, these marginal benefits will differ across people. With a private good, people with a strong taste for a good will consume higher quantities of that good, if the allocation is efficient. The high quantities they consume drive down their MRS, to equal that of other people. But with a pure public good, everyone consumes the same quantity. A person with a strong taste for that public good will have a higher MRS than someone with a weaker taste. But as long as both people have a positive marginal benefit from the good, it is efficient to let them both consume all the quantity which is available.

Adding up Demand Curves Vertically

A typical demand curve has the price of the good on the vertical, and the quantity consumed on the horizontal. Even if a good is non–rival and non–excludable, people's demand curves for the good can be drawn, even if a market for the good cannot (and should not) operate very well.

There are two ways of interpreting a demand curve. The usual interpretation is that the price (on the vertical axis) is the independent variable and the quantity (on the horizontal axis) is the dependent variable. Under that interpretation, a person's demand curve for a good shows how much of the good the person would be willing to purchase, as a function of the price she has to pay for the good. This interpretation still makes sense for pure public goods : we can ask how many hours of television programming a person would want to buy, if she had to pay \$10 per hour to watch. We can ask this question, and draw the demand curve from the answers, even if the person does not actually have to pay for the television programming, and even if there is no way that we could make her pay.

The second interpretation of the demand curve is the "inverse demand curve" in which the height of the demand curve is interpreted as the dependent variable. That is, the height of any demand curve represents how much the person would be willing to pay for a little more of the good. Note that both interpretations are correct. If $(10, 1.2)$ is a point on a person's demand curve for potatoes, then both of the following statements are true : (1) she would choose to buy 10 kilograms of potatoes per year if the price of potatoes were \$1.2 per kilogram ; (2) if she were consuming 10 kilograms of potatoes per year, then the most she would be willing to pay for a small increase in potato consumption is \$1.2 per kilogram.

So a person's willingness to pay for a little more of the pure public good is the height of her demand curve for the public good. The Samuelson condition is that the sum of the MRS's of the different people should equal the MRT. Since the MRS's are the heights of the people's demand curves for the public good, then the Samuelson condition means that the demand curves for the public good should be added vertically, and that this vertical sum should be set equal to the marginal rate of transformation, which would be the height of the supply curve (if the public good were supplied by competitive firms).