Lindahl Prices

So far, the efficient allocation in the presence of public goods has been characterized in two ways :

a an allocation for which the sum of people's MRS's equals the MRT

b the level of public good provision at which the sum of the **heights** of people's demand curves equals the marginal cost of public good provision

These two characterizations are pretty similar : the MRS, or willingness to pay, is just the height of a demand curve. But there are some differences between the two approaches. The second approach can be interpreted as using ordinary (that is, **uncompensated**) demand curves. With ordinary demand curves, the height of each person's demand curve depends on the person's income.

To help distinguish between these approaches, the second approach, using the uncompensated inverse demand curves, will now be referred to as **Lindahl** pricing, after the Swedish economist who first proposed it.

The rule for Lindahl pricing can be written mathematically as

$$P_z^1[Z, M_1] + P_z^2[Z, M_2] = c$$

where $P_z^1[Z, M_1]$ is person 1's inverse demand function, expressing her willingness to pay for the public good as a function of the quantity Z of the public good, and her income M_1 (and perhaps on some other factors as well). Since P_z^i is the height of person i's demand curve for the public good, it is a price that applies **only to person** i. As long as people's tastes differ, then one person's demand curve will look different from another's. Even with everyone consuming the same **quantity** Z of the public good, the willingness to pay will differ among people. The price P_z^1 is referred to as person 1's Lindahl price for the public good.

This Lindahl price is a personalized price : potentially there is a different price for each person in the country, even though they all are consuming the same quantity of the public good. With ordinary private goods, people all face the same price for the good ; they then choose the quantity they wish to consume, and these quantities will differ among people. With a pure public good, the opposite situation arises : everyone consumes the same quantity of the good, but people's "price"s are different.

Why is "price" in quotation marks? The price of a private good is a price that people actually face : they can buy the good for that price in private markets. The Lindahl price is a **hypothetical price** ; it is the price that a person would be willing to pay for a little more of the good, if someone were to offer her a little more of the good.

But Lindahl prices could be used to finance the pure public good. Consider the Cobb–Douglas example (of subsection #3 in this section), in which person 1's and 2's preferences were represented by the utility functions

$$U^1(x_1, z_1) = 2 \ln x_1 + \ln z_1$$

$$U^2(x_2, z_2) = \ln x_2 + 2 \ln z_2$$

It then follows that

$$P_z^1 = \frac{1}{Z}\frac{1}{3}M_1$$

$$P_z^2 = \frac{1}{Z}\frac{2}{3}M_2$$

[Where did those inverse demand functions come from? See the appendix to chapter 5 in Varian's *Intermediate Microeconomics*, or example 4.3 in Nicholson's *Microeconomic Theory.*]

Suppose as well that the equation of the production possibility frontier was $Z + X = 120$, so that $c = 1$.

Then the Lindahl solution is to find the level Z of public good provision such that $P_z^1 + P_z^2 = c$, or the solution to the equation

$$\frac{1}{Z}(\frac{M_1}{3} + \frac{2M_2}{3} = 1$$

which is the same as

$$Z = \frac{M_1}{3} + \frac{2M_2}{3}$$

If $M_1 = 90$ and $M_2 = 30$, then the Lindahl solution, the level of Z for which the vertical sum of the demand curves equals 1, is $Z = 50$. When $M_1 = 90$, $M_2 = 30$ and $Z = 50$, then $P_z^1 = 0.6$ and $P_z^2 = 0.4$. The fact that $P_z^1 + P_z^2 = 1$ shows that this is a Lindahl solution.

Here the cost of providing a level Z of the public good is simply Z, since here $c = 1$. One way of **financing** the public sector would be to "charge" each person a bill equal to her or his consumption of the public good, times her or his personalized price P_z^i. Person 1 would face a bill of $(0.6)(50) = 30$, since she consumes 50 units at a personalized price of 0.6 each. Person 2's bill would be 20, so that the sum of each person's payments exactly equals the cost of the public sector.

The fact that these people's payments, using the height of each person's demand curve as a personalized price, exactly cover the cost of the public sector, is not a coincidence. It follows from the fact that the prices must sum to c for the allocation to be a Lindahl solution, and from the assumption that the unit cost of the public good is a constant c.

Advantages of Lindahl Pricing

The Lindahl solution involves both the level of public good provision, and the method by which it is financed. The method of finance is that each person's tax bill equals the quantity of public goods consumed, times the price per unit. But each person's price will be different.

This solution is sometimes called "benefit taxation", since each person's Lindahl price is the marginal benefit she derives from the good being financed. The Lindahl solution covers the cost of the public sector (and more than covers it if the marginal cost of the public good is increasing). It yields an efficient allocation. It also charges people proportionately to some measure of the benefit they derive from the public good being financed. If one person's demand curve is above another person's — indicating the first person has a higher demand for the public good — then this first person will pay more taxes to finance the public good.

Another attractive feature of the Lindahl solution is that it leaves everyone better off than they would be without the public good. Each person gets a consumer surplus, equal to the area beneath her demand curve, above her Lindahl price.

That would not be necessarily be the case, if we financed an efficient allocation using some other tax scheme, rather than Lindahl pricing. Figure 1 illustrates. In this example, the MRT is 20, and given the inverse demand curves of the two people, the sum of their MRS's equals 20 at a public good level of 10. The total consumer surplus received by person 2 is 80 when $Z = 10$, and the total consumer surplus received by person 1 is 245. At $Z = 10$, person 2's willingness to pay is 5 and person 1's is 15. So Lindahl pricing would mean that person 2 paid $(5)(10) = 50$ and person 1 paid $(15)(10) = 150$; each person's Lindahl tax bill is less than her or his total consumer surplus. On the other hand, if the government simply split the total cost of the public good [which is 200 here] between the two people, then person 1's tax bill would be 100, which is more than the consumer surplus she receives.

Another apparent advantage of Lindahl pricing is the parallel with competitive markets for private goods. In competitive markets for private goods, each person's total expenditure on the good is her marginal willingness to pay (the height of her demand curve) times the quantity which she consumes. In an economy with private goods, competitive markets yield an efficient allocation of goods and services. Therefore, there is some appeal to a procedure for provision of public goods which seems to mimic the way competitive markets for private goods work. However, as will be discussed below, there is an important difference between a competitive market for private goods and the "pseudo–market" for public goods represented by the Lindahl solution.

Drawbacks to Lindahl Pricing

There are some drawbacks to Lindahl pricing. Although using Lindahl prices to finance the cost of some public good is often called "benefit pricing", these prices are *marginal* benefits, not total benefits. In the example in figure 1, the marginal and total benefits each person gets at the Lindahl solution are roughly proportional to each other. (Person 2's marginal and total benefits when $Z = 10$ are each roughly 3 times person 1's.) But this need not be the case, as figure 2 illustrates. Here person 1's demand curve starts out much above person 1's, but is also much steeper than person 2's. That is, person 1 has a much less elastic demand than person 2. In this example, the efficient level of public good provision is 9.5. At $Z = 9.5$, person 1's marginal willingness to pay is 2.5, and person 2's is 19.1, so that the sum of the heights of their demand curves does equal the MRT, which is 21.6 in this example. But even though person 1's *marginal* willingness to pay is much lower than person 2's, her *total* willingness to pay is actually higher, since she values the first few units of the public good so much. (In this example, the area under person 1's demand curve, up to $Z = 9.5$, is about 249 and the area under person 2's is only about 191)

A bigger problem with Lindahl pricing is the availability of the relevant information. The great advantage of competitive markets for private goods is that no firm needs to know anything about the tastes of individual consumers. (And consumers do not have to know anything about the production technology of firms.) The only thing a competitive firm has to know — other than its own production technology — is the price at which it can sell the good. The only things a consumer has to know — other than her own tastes — are the prices of the goods and services available. This is an enormous advantage of competitive private markets. Recall that the first and second fundamental theorems of welfare economics (Nicholson chapter 17, Varian chapters 31 and 32) imply that the allocation resulting from perfect competition is Pareto optimal. That means that perfect competition is "just as good" as what a benign social planner could achieve. But the big advantage of private markets over central planning is that a benign social planner would have to know everyone's tastes, and all firms' technologies, in order to implement a Pareto optimal allocation. Under perfect competition, there is no need for anyone to learn this information, and therefore no problem in getting people to reveal their preferences or firms to reveal their technologies.

If a government (or anyone else) wants to implement Lindahl pricing, it must know people's demand curves for the public good. Thus the key difference between markets for private goods, and "pseudo–markets" for public goods, is that more information is required to implement an efficient allocation when there are public goods.

If a good is nonrival, but excludable, there is the possibility of learning people's demand curves from observing what they buy, if they are given the opportunity to buy the good at different prices. But even here, there are problems which do not arise with private goods. Even if a nonrival good actually is excludable, it is not efficient to actually exclude anyone from consumption. So if we wanted an efficient allocation, we might use exclusion as an experiment : sell the good at various prices, learn people's demand functions, and then use this information to provide an efficient quantity of the public good (which would then be made available to everyone without exclusion). The problem is : knowing that the results of this experiment would be used in determining the quantity of public goods to provide, and how much each person would be paying, would influence people's behaviour. If I thought that the government was going to use benefit taxation, and was making this excludable nonrival good available at different prices in order to learn my own demand curve, then I might buy less than I really wanted, in the hope that this will get me a lower tax bill when benefit taxation was introduced.

The basic problem, then, with the Lindahl solution, may be getting people to reveal their preferences. This problem is not unique to Lindahl pricing : with public goods, to find any efficient allocation, people's marginal rates of substitution must be learned.