

1. The efficiency condition for provision of a public good is that the sum of people's *MRS*'s equal the *MRT*.

In this question, the production possibility is a straight line with slope -1 , so that the *MRT* equals 1.

A person's *MRS* is the ratio of her marginal utilities. Person 1 has a utility function

$$U^1(x_1, Z) = x_1 + 2\sqrt{Z}$$

so that

$$MU_x^1 = 1 \quad MU_Z^1 = \frac{1}{\sqrt{Z}}$$

implying

$$MRS^1 = \frac{MU_Z^1}{MU_x^1} = \frac{1}{\sqrt{Z}}$$

Person 2 has the utility function

$$U^2(x_2, Z) = x_2 + 4\sqrt{Z}$$

so that

$$MU_x^2 = 1 \quad MU_Z^2 = \frac{2}{\sqrt{Z}}$$

and

$$MRS^2 = \frac{MU_Z^2}{MU_x^2} = \frac{2}{\sqrt{Z}}$$

The efficiency condition therefore is

$$\frac{1}{\sqrt{Z}} + \frac{2}{\sqrt{Z}} = 1$$

or

$$\sqrt{Z} = 3$$

meaning that efficiency requires a level of public good provision of $Z = 9$.

Any feasible allocation (x_1, x_2, Z) with $Z = 9$ — that is, any allocation (x_1, x_2, Z) with $Z = 9$ and $x_1 + x_2 + Z = 15$ and $x_1 \geq 0$, $x_2 \geq 0$ will be efficient.

So the feasible allocations are the combinations (x_1, x_2, Z) with $Z = 9$, and with $x_1 + x_2 = 6$, $0 \leq x_1 \leq 6$.

2. If a public good is provided through voluntary provision, the equilibrium contributions will be inefficiently low if people care only about their own utility, and behave non-cooperatively. However, the contributions will not be zero in general.

The equilibrium would involve no contributions at all only if

$$MRS^i(x_i, Z) \leq c \quad \text{when} \quad x_i = M_i, \quad Z = 0$$

for each person i , where MRS^i is person i 's marginal rate of substitution of the public good for the private, a function of her private good consumption x_i and the total quantity Z of the public good.

Otherwise there will be a Nash equilibrium in which some people do make contributions, but only up the level ζ_i of contributions such that

$$MRS^i(M_i - \zeta_i, Z) = c$$

when

$$Z = \sum_j \zeta_j$$

3. If there is a positive externality, then some activity of firm 1 will benefit firm 2. The efficient level of that activity is the level such that

$$MB_2^1 + MB_1^2 = MPC$$

where MB_1^1 is the marginal benefit to firm 1 of a little more of the activity, MB_1^2 is the marginal benefit to firm 2 of a little more of firm 1's activity and MPC is the marginal cost of the activity.

If firm 1 acts on its own, it will consider only its own private benefits, and choose an activity level such that

$$MB_1^1 = MPC$$

resulting in *too low* a level of the activity (since the MPC curve slopes up, the MB_1^1 curve slopes down, and since $MB_1^2 > 0$ if there is a positive externality).

Whether government intervention is needed to achieve an efficient level of the activity depends on whether the firms are able to negotiate a solution. If they are able to negotiate, firm 2 would pay firm 1 to increase the level of the activity to the efficient level.

If the firms do not negotiate a solution the government can achieve efficiency by ordering firm 1 to increase the activity to the efficient level, or could subsidize the activity, with the subsidy rate equal to the external benefit to firm 2.