AS/ECON 4080   Answers to Mid–term   February 2002

1. Suppose that good Z is a non–rival good, and good X is a rival good. Then if the allocation is efficient, it must be true that the sum of all people's marginal rate of substitution between Z and X must equal the marginal rate of transformation between Z and X. That is,

$$MRS^1_{ZX} + MRS^2_{ZX} + \cdots MRS^N_{ZX} = MRT_{ZX} \qquad (*)$$

if MRS^i_{ZX} is person i's marginal rate of substitution of the non–rival good for the rival good, and if there are N people in the economy, and if MRT_{ZX} is the marginal rate of transformation.

If goods Y and X are both rival goods, then efficiency requires that

$$MRS^1_{YX} = MRS^2_{YX} = \cdots = MRS^N_{YX} = MRT_{YX} \qquad (**)$$

An allocation will be efficient if it is feasible, if total production is on the production possibility frontier, if equation $(*)$ holds for any pair of goods where one of them is non–rival and the other is rival, and if equation $(**)$ holds for any pair of rival goods.

Equation $(*)$ is the same as the condition that the quantity provided of the non–rival good Z is the quantity such that the *vertical* sum of people's demand curves for Z equals the marginal cost of producing Z. Alternatively, the condition is the same as the sum of each person's willingness to pay for the non–rival good (in terms of foregoing some of the rival good X) equalling the marginal cost of the non–rival good.

Efficiency requires condition $(*)$ to hold because if a little more of the non–rival good is produced, each person will consume a little more. Therefore, the marginal benefit of producing a little more is the sum of each person's marginal benefit. (In contrast, a unit of a rival good can be consumed by at most one person, so that the marginal benefit of producing a little more of a rival good is the benefit to the person who gets to consume it.)

2. One tax mechanism which will induce people to reveal truthfully how much they value a public project relies on a "pivot tax" in addition to taxes which might pay for the cost of the project.

Suppose that there are N people who will use the project. Let v_i represent how much person i is truly willing to pay for the project. Let w_i denote the amount that the person *says* that she is willing to pay for the project. Let C denote the total cost of the project. Then a mechanism which would induce people to reveal truthfully their willingness to pay for the project has these rules :

A : the project will be undertaken if, and only if, the sum of people's announced willingness to pay exceeds the cost of the project, that is, if and only if

$$w_1 + w_2 + \cdots w_N > C$$

B : if the project is undertaken, each person will pay a tax of C/N to pay for the cost of building the project

C : in addition, person i will have to pay an extra pivot tax under some circumstances ;

$C1$: if $\sum_{j \neq i} w_j > \frac{N-1}{N} C$, and as well $\sum_{j=1}^{N} w_j \leq C$, then person i has been pivotal in getting the project rejected, and will have to pay an extra tax t_i of

$$\sum_{j \neq i} w_j - \frac{N-1}{N} C$$

$C2$: if $\sum_{j \neq i} w_j \leq \frac{N-1}{N} C$, and as well $\sum_{j=1}^{N} w_j > C$, then person i has been pivotal in getting the project approved, and will have to pay an extra tax t_i of

$$\frac{N-1}{N} C - \sum_{j \neq i} w_j$$

Given these taxes, if a person, for example, overstates her willingness to pay for the project, then she runs the risk of being pivotal in getting the project approved, and having to pay the extra tax defined in rule $C2$. If she overstated her willingness to pay, then she could wind up paying more in taxes than the project is worth to her. By revealing truthfully her willingness to pay for the project, she ensures that she would be pivotal in getting the project approved only if the value of the project to her exceeds the taxes she might have to pay. (Similarly, she ensures that she would be pivotal in getting the project rejected only if the pivot tax $C1$ she would have to pay is less than the cost to her of having the project approved.)

3. Since person 1's cigarette smoking causes an externality to person 2, the allocation will be efficient only if the marginal benefit of one more cigarette to person 1 equals the marginal cost of the cigarette plus the marginal damage that the cigarette smoking imposes on person 2.

In this case, the equation of the production possibility frontier is $x_1 + x_2 + z_1 = 100$, so that the PPF has a slope of -1, and the marginal cost of one more cigarette (in terms of foregone food) is 1. The marginal benefit to person 1 of another cigarette (in terms of food) is her marginal rate of substitution of cigarettes for food, so that

$$MB_1 = \frac{MU_z^1}{MU_x^1}$$

If $U^1(x_1, z_1) = x_1 + 21z_1 - z_1^2$, then

$$MU_x^1 = 1$$

and

$$MU_z^1 = 21 - 2z_1$$

so that

$$MB_1 = 21 - 2z_1$$

The marginal damage done to person 2 from person 1's smoking (measured in terms of food), is her marginal disutility from cigarette smoking, divided by her marginal utility of food. Since $U^2(x_2, z_1) = x_2 - z_1^2$,

$$MU_x^2 = 1$$

and

$$MU_z^2 = -2z_1$$

so that the marginal damage is

$$MD = 2z_1$$

An efficient allocation requires that $MB_1 = MC + MD$, or

$$21 - 2z_1 = 1 + 2z_1$$

so that in any efficient allocation

$$z_1^* = 5$$

The efficient allocations are all allocations (x_1, x_2, z_1) such that $z_1 = 5$, $x_1 + x_2 = 95$, and $x_1 \geq 0$, $x_2 \geq 0$.