

1. When there is one private good and one public good, the efficient allocations are all the allocations which : *i* are efficient in production ; *ii* allocate non-negative quantities of each good to each person ; *iii* allocate the same quantity of the public good to each person and *iv* satisfy the Samuelson condition

$$\sum MRS^i = MRT$$

In this question, efficiency in production means that the aggregate production plan must be on the production possibility curve, or

$$x_1 + x_2 + Z = 300 \tag{1}$$

Here, given the utility functions,

$$MRS^1 = MU_Z^1 / MU_X^1 = \frac{20}{Z}$$

$$MRS^2 = MU_Z^2 / MU_X^2 = \frac{30}{Z}$$

(where I have incorporated the requirement *iii*, that $z_1 = z_2 = Z$). Therefore, the Samuelson condition is

$$\frac{20}{Z} + \frac{30}{Z} = 1 \tag{2}$$

since the slope of the production possibility frontier is -1 . Therefore, the Samuelson condition implies

$$Z = 50$$

Any allocation (x_1, x_2, Z) such that $Z = 50$, and $x_1 + x_2 = 250$, with $0 \leq x_1 \leq 250$ will be efficient.

2. The Nash equilibrium, if each person acts independently, and in her own self-interest, is a list of contribution levels $(\zeta_1, \zeta_2, \dots, \zeta_{100})$ by the people, with a total public good level of $Z = \zeta_1 + \zeta_2 + \dots + \zeta_{100}$, such that :

ζ_i equals 0 if $MRS^i(m_i, Z) < 1$, where MRS^i is the person's marginal rate of substitution of the public good for the private good, and m_i is the person's income (these are functions of the person's private good consumption, and of her public good consumption), and such that :

the person contributes an amount $\zeta_i > 0$ for which $MRS^i(m_i - \zeta_i, Z) = 1$ if $MRS^i(m_i, Z) \geq 1$

The MRS is the person's willingness to pay. So what we can conclude is that, at the equilibrium, the first 10 people (who made positive contributions) have MRS 's equal to 1, and the other 90 people (who chose not to contribute) have a willingness to pay which is less than 1.

What would happen if a person gave a more, and if each of the other 99 people also gave the same a ? Then the total quantity of the public good would go up by $100a$, and the person's own

private good consumption would go down by the amount she contributed, a . The net change in her utility would be

$$-MU_x a + 100MU_Z a = MU_x(100MRS - 1)a$$

since the MRS is the ratio of the marginal utilities MU_Z and MU_x of a little more public good consumption and private good consumption respectively.

So as long as her MRS is greater than $1/100$, she would be made better off by this kind of “matching” deal, in which she gives some money but everyone else matches her contribution. The first 10 people, who chose to make positive contributions in equilibrium, each have an MRS of 1, so they certainly would agree to such a deal. The other 90 people have MRS ’s somewhere between 0 and 1 ; so they would agree to the deal if their MRS was $1/100$ or more.

3. The statement is an exaggerated version of the “Coase theorem”, that “property rights don’t matter”. If the parties to an externality can negotiate cheaply, then it will not matter for the efficiency of the outcome. That is, whether or not the firm is allowed to pollute, negotiation would lead to an efficient outcome. If the firm has the right to pollute, then the “victim” of pollution would have an incentive to bribe the first firm to reduce pollution to the efficient level. If the firm has no right to pollute, then it would have an incentive to bribe the “victim” to allow it to pollute up to the efficient level.

Who has the property right (that is, whether the firm is allowed to pollute or not) certainly **does** matter for the distribution of profits between the firms. The first case (in which the firm has the right to pollute) means more profit for the polluting firm, and less for the “victim” than the second case.

If some of the parties to the externality are people, not firms, then the distribution of the property rights may actually affect the level of the externality. But — providing negotiation is easy — the allocation will be efficient whatever is the distribution of property rights.

If negotiation is not so easy, then the “correct” bribes might not be made, and the distribution of property rights will matter for the efficiency of the outcome.

In summary, the statement is “true”, but only under certain assumptions. A more precise statement would be :*If negotiation is easy, then it will not matter at all for the efficiency of the outcome whether a firm is allowed to pollute as much as it wishes, or whether it cannot pollute at all without the permission of all affected parties*”.