AS/ECON 4080 Answers to Mid–term Exam February 2005

$Q1$. Suppose that for some people (but not for all people) there was some maximum amount $\bar{Z}$ of a public good which they wished to consume. For quantities above $\bar{Z}$, these people receive no additional benefit.

Would it ever be efficient to provide a quantity of this public good which is greater than $\bar{Z}$? Explain.

$A1$. When there is a public good, efficiency requires that the Samuelson condition hold,

$$MRS^1_{ZX} + MRS^2_{ZX} + \cdots + MRS^n_{ZX} = MRT_{ZX}$$

where MRS^i_{ZX} is person i's marginal rate of substitution between the public good, and some private good X, and where MRT_{ZX} is the marginal rate of transformation between the public good and the same private good X.

According to the question, $MRS^i_{ZX} = 0$, when $Z > \bar{Z}$, for some people. Suppose that the people who get no extra benefit from the public good (for $Z > \bar{Z}$ are people numbers $1, 2, \ldots, m$, and that people numbers $m+1, m+2, \ldots, n$ all did still receive positive marginal benefits from the public good, even if $Z > \bar{Z}$.

Then there would be an efficient allocation in which the quantity of public goods supplied was $Z^* > \bar{Z}$ whenever

$$MRS^{m+1}_{ZX} + MRS^{m+2}_{ZX} + \cdots + MRS^n_{ZX} = MRT_{ZX}$$

at some $Z^* > \bar{Z}$, or equivalently, if

$$MRS^{m+1}_{ZX} + MRS^{m+2}_{ZX} + \cdots + MRS^n_{ZX} > MRT_{ZX}$$

at $Z = \bar{Z}$.

This is certainly possible, if the taste of people $m+1, m+2, \ldots, n$ for the public good is strong enough.

So the answer is "yes", it could well be efficient to provide a quantity of the public good greater than $\bar{Z}$.

$Q2$. Describe briefly a tax mechanism which would induce people to reveal truthfully how much they are willing to pay for a single "all or nothing" public project.

$A2$. An example of such a mechanism is the mechanism described in note #1 of the lecture notes on "preference revelation and the free rider problem".

Suppose that there are n people, and that the cost of the project is C. In this mechanism, each person i is asked to announce an amount v_i that she is willing to pay to have the project built. The answers to this survey are then used in the following way *i* the project is built only if the sum of the announced valuations is at least as large as the cost of the project, that is if and only if

$$v_1 + v_2 + \cdots + v_n \geq C$$

ii if (and only) the project is built, each person has to pay a tax of C/n to cover the cost of building it

iii in addition, each person may be liable for an additional "pivot tax", t_i, calculated in the following manner

iiia if $v_2 + \cdots + v_n < \frac{n-1}{n}C$, but $v_1 + v_2 + \cdots + v_n \geq C$, then person #1 is liable for a pivot tax of

$$t_1 = \frac{n-1}{n}C - (v_2 + v_3 + \cdots + v_n)$$

iiib if $v_2 + \cdots + v_n \geq \frac{n-1}{n}C$, but $v_1 + v_2 + \cdots + v_n < C$, then person #1 is liable for a pivot tax of

$$t_1 = (v_2 + v_3 + \cdots + v_n) - \frac{n-1}{n}C$$

iiic every other person, $2, 3, 4, \ldots, n$ is liable for pivot taxes if he or she is pivotal, with the taxes defined analogously to those in *iiia* and *iiib*

If the government can commit to using these rules, then each person will find it in her own self–interest to announce her own best estimate of what she really does think the project is worth to her. Because of the pivot taxes, there is no advantage to lying, and there may be a disadvantage. For example, if person #1 had a high true value for the project, understating that value would matter only if the other $n - 1$ people had values which averaged to a number close to, but less than, C/n. In such a case, the added pivot tax person #1 might have to pay, if her true answer led to the project being undertaken (that is, if she were pivotal), would be less than the net benefit to her of having the project built.

*Q*3. Suppose that firms 1 and 2 both sell their outputs on competitive markets, at a price of \$1 per unit of output sold. Suppose as well that each firm could hire labour at a wage of \$10 per hour, and buy coal at \$10 per tonne.

Firm 1's output, as a function of its use L_1 of labour and Z_1 of coal, is

$$F^1(L_1, Z_1) = \ln L_1 + 210Z_1 - (Z_1)^2$$

while firm 2's output, as a function of its own labour use L_2, and firm 1's coal use Z_1, is

$$F^2(L_2, Z_1) = \ln L_2 - (Z_1)^2$$

(where "ln" refers to the natural logarithm function).

How much coal would firm 1 use, if it had to compensate firm 2 for any damage its coal use imposed, **if the two firms were able to negotiate with each other**?

*A*3. If the firms can negotiate with each other, then the outcome will be the efficient one (regardless of the rules imposed by the courts). In this case, firm #1 would pay firm #2 in order to be able to use some coal, and these payments would lead to the firms negotiating an agreement in which the efficient level of coal was used.

What is the efficient level? Efficiency requires that

$$MB^1 = w_Z + MD^2$$

where MB^1 is firm #1's marginal private benefit from using coal, w_Z is the price of coal, and MD^2 is the marginal damage done to firm #2.

Here

$$MB^1 = p_1 \frac{\partial F^1}{\partial Z_1} = 210 - 2Z_1$$

$$w_Z = 10$$

$$MD^2 = -p_2 \frac{\partial F^2}{\partial Z_1} = 2Z_1$$

so that efficiency requires that

$$210 - 2Z_1 = 10 + 2Z_1$$

or

$$Z_1 = 50$$

Because of the separability of the production functions, the levels of labour used by the firms does not actually affect the efficient level of coal used in this example. The efficient levels of labour are the solutions to

$$p_1 \frac{\partial F^1}{\partial L_1} = w_L$$

$$p_1 \frac{\partial F^1}{\partial L_1} = w_L$$

Here, those equations are

$$\frac{1}{L_1} = 10$$

$$\frac{1}{L_2} = 10$$

so that the efficient levels of labour use by each firm are $L_1 = L_2 = \frac{1}{10}$.