AS/ECON 4080 Answers to Mid–term Exam February 2006

$Q1$. Suppose that good X is a pure private good, and that good Z is a **non–rival** good which is also completely **excludable** at zero cost. Discuss the conditions for efficiency in an economy with these 2 goods.

$A1$. What matters for efficiency is whether a good is rivalrous or non–rivalrous, not whether or not it is excludable. If a good is non–rival, then consumers should not be excluded from its benefits, even if exclusion is costless.

So if good X is a rival good, and if good Z is a non–rival good, then an allocation is efficient if and only if the following conditions hold :

(i) aggregate production is efficient : (X, Z) is on the production possibility frontier, if X is aggregate production of the rivalrous good and Z is aggregate production of the non–rivalrous good

(ii) $z_1 = z_2 = \cdots = Z$, where z_i is person i's consumption of the non–rival good ; people should not be excluded from benefits of the non–rival good

(iii) $MRS^1_{ZX} + MRS^2_{ZX} + \cdots + MRS^n_{ZX} = MRT_{ZX}$, where MRS^i_{ZX} is person i's marginal rate of substitution of the non–rivalrous good for the rivalrous good — her willingness to pay, in units of the rivalrous good, for a little more of the non–rivalrous good — and MRT_{ZX} is the marginal rate of transformation — the cost, in foregone units of the rivalrous good, of a little more production of the rivalrous good

$Q2$. How much tax revenue would be collected by a "pivot tax" mechanism, which induced people to reveal their preferences truthfully, in the following situation?

The indivisible ("all or nothing") public project costs \$1000. There are 5 people ; 3 people value the project at \$300 each, and 2 people value the project at \$30 each. The project will be undertaken if and only if the sum of people's announced valuations exceeds

the cost of the project, \$1000. If the project is undertaken, each person will pay the same share, \$200, of the cost. In addition, if any person is "pivotal" (that is, if her valuation alters the overall result), then she will have to pay a pivot tax, equal to the (absolute value of the) difference between the sum everyone else's announced valuations and the sum of the shares of the cost which they must pay.

*A*2. A pivot tax mechanism will induce people to reveal truthfully their willingness to pay for the public good, if they understand the mechanism. Here, the sum of the 5 peoples' willingness to pay for the project is $300 + 300 + 300 + 30 + 30 = 960 < 1000$, so that the project will not be built if people respond truthfully when asked their valuations.

The two "llow–value" pople here are pivotal. If the three high–value people, and one of the two low–value people are asked their valuations, and if they respond truthfully, then the four people's total valuations will sum to $300 + 300 + 300 + 30 = 930$, which exceeds their share of the costs, 4/5 of 1000, which is 800. Therefore a person with a valuation of 30 is pivotal : without this person's response, the average vbaluation of the other 4 people is $930/4 = 232.50$, which exceeds the cost per person of the project, 200, but with the person's response the average valuation is $960/5 = 191$, which is less than the average cost.

That means that a person whose value was \$30 would have to pay a pivot tax, equal to the difference of everyone else's announced benefits, \$930, and their share of the costs, \$800. The pivot tax collected from this person would be $930 - 800 = 130$.

Since there are two people with valuations of 30, both of them are pivotal, and 130 in pivot tax would be collected from each, for a total pivot tax revenue of 260.

Note that these low–value people would still prefer to tell the truth : revealing their low value makes them each liable for a pivot tax of 130, but exaggerating their announced valuation, so that they are not pivotal, and the project does get built, would make them liable for their share of the cost of the project, \$200, giving them a net benefit of $30 - 200 =$

-170, which is a worse outcome than paying the pivot tax.

here, none of the three high–value people are pivotal. When a high–value person's response is not included, the project won't be built. But even when the response is inclued, the sum of the announced valuations still is less than the cost. So the response of an individual high–value person does not change the outcome, so that none of these three people is pivotal.

*Q*3. What government action, if any, would be needed if firm A and B share access to the same harbour for shipping, and if firm A's own investment in improvements to the port facilities also benefitted firm B?

*A*3. This is an example of a positive externality. If I_A is firm A's investment in port improvements, then efficiency requires a level of investment such that

$$p_A MP_I^A + p_B MP_I^B = w_I \tag{3 - 1}$$

where p_i is the price of firm i's output, MP_I^i is the marginal productivity of port improvements on the output of firm i (that is, the increase in firm i's output caused by an increase in I_A), and w_I is the unit cost of the improvements.

If firm A were to ignore the effects of its own investments on firm B's profits, then it would choose an investment level such that

$$p_A MP_I^A = w_I$$

resulting in too little investment in port improvements, since it is ignoring one of the benefits of its own investments, the increase in the other firm's profits.

Possible government–imposed remedies would be a subsidy to firm A's port improvement, with the unit subsidy equal to $p_B MP_I^B$ (evaluated at the efficient level of investment I_A^*, the level which satisfies $(3-1)$), direct government regulation, in which firm A is ordered to invest at least I_A^* in port improvements, or fines on firm A if its investment is less than I_A^* (with the unit fine equal to MP_I^B).

But government intervention is required only if the firms cannot resolve the externality themselves. If negotiation is easy, they should be able to do so. The two firms might merge, which would lead the new merged firm to take account of all of the effects of the port improvements. Firm B might bribe firm A to increase its own port improvement investment ; there is always some bribe which would induce firm A to invest I_A, and would make both firms better off than they would be if firm A simply ignored firm B.