AP/ECON 4080 Answers to Mid–term Exam February 2012

$Q1$. If the cost of producing one unit of a pure public good is \$1, and the cost of producing one unit of a pure private good is \$1, what are the efficient allocations in an economy with 1000 people, if each person's preferences can be represented by the utility function

$$U(x, z) = x + \ln z$$

where x is the person's private good consumption, and z is her public good consumption (and "ln" represents the natural logarithm), if the total income in the economy (available for public and private good provision) is \$3000?

$A1$. The "Samuelson condition" for an efficient allocation when there are pure public goods is :

$$\sum MRS = MRT$$

Here the marginal rate of transformation (MRT) is 1, since the unit cost of the public good is \$1 and the unit cost of the private good is \$1 (the MRT is the ratio of the marginal costs of the goods).

A person's MRS is the ratio of her marginal utilities, or

$$MRS = \frac{\partial U/\partial z}{\partial U/\partial x}$$

Since the derivative of the natural logarithm of z is $1/z$, here

$$MRS = \frac{1}{z}$$

for each person. Since this is a pure public good, non–rivalry means that each person's public good consumption z will be the same, and will equal the level Z of the public good provided to the whole economy. Since there are 1000 people, the Samuelson condition is

$$1000\frac{1}{Z} = 1$$

or

$$Z = 1000$$

The only efficient allocations here are those which provide exactly 1000 units of the pure public good. The economy has a total income of \$3000. So, after providing the public good — 1000 units at \$1 per unit — there will be \$2000 left over for private goods. Therefore, **any** allocation which gives a total private good consumption of 2000 will be efficient. If x_i is the private good consumption of person i, then an allocation will be efficient if and only if :

$$x_i \geq 0 \quad \text{for} \quad \text{each} \quad \text{person} \quad i$$

$$x_1 + x_2 + \cdots + x_{1000} = 2000$$

$$z_1 = z_2 = \cdots = z_{1000} = 1000$$

*Q*2. How much tax revenue would be collected by the following "pivot tax" mechanism, if each person tries to use the mechanism to make herself as well off as possible?

The indivisible ("all or nothing") public project costs \$1000. There are 4 people : each person knows how much she values the project (but nobody else knows her valuation). Person #1 values the project at \$600, person # values it at \$200, person #3 values the project at \$200, and person #4 values it at \$200.

The rules of the tax are : the project will be undertaken if and only if the sum of people's announced valuations exceeds the cost of the project, \$1000. If the project is undertaken, each person will pay the same share, \$250, of the cost. In addition, if any person is "pivotal" (that is, if her valuation alters the overall result), then she will have to pay a pivot tax, equal to the (absolute value of the) difference between the sum everyone else's announced valuations and the sum of the shares of the cost (750) which they must pay.

*A*2. A person will pay a pivot tax, on top of her share of the cost (if the project in undertaken) only if she is pivotal. She would be pivotal if her announced valuation switched the average of all announced valuations from less than \$250 without her to more than \$250 with her included, or vice versa.

If people understand the mechanism, then each person will realize the best strategy is to reveal truthfully his or her valuation of the project.

If people announce their true valuations, the sum of their valuations is $600+200+200+200 = 1200 > 1000$ and the project will be built : the average of all 4 people's valuations is 300.

But three of the people (people #2, #3, and #4) have valuations less than the threshold level of \$250. So here person 1 — and only person 1 — is pivotal. The average valuation with her included is $300 > 250$ but the average valuation with her excluded is $200 < 250$.

So, if everyone announce her or his true valuations, person #1 would have to pay a pivot tax equal to the difference between 750 and the sum of other people's announced valuations, which here is 600.

So if every person chose her or his best strategy, the outcome would be : the project is undertaken ; every person pays \$250 to cover the cost of the project ; person #1 pays a pivot tax of \$150.

[Note that this extra pivot tax revenue will **not** be paid as compensation to people #2, #3 and #4.]

[Also note that,even if she could foresee the outcome, person #1 would not want to lie about her strong taste for the project. If she announced, for example, a valuation of \$200, she would avoid paying any pivot tax, and she would not have to pay her share of the project's cost, because the project would not be undertaken. But that strategy would save her \$400 in total taxes — while losing her a project which she values at \$600. Better to pay the taxes and get the project built.]

*Q*3. If firm #1 uses flour as an input to production, and if firm #2's output is a decreasing function of firm #1's flour usage (since flour particles clog up firm #2's machinery), give **three** different remedies which ensure that firm #1 chooses its quantity of flour efficiently.

*A*3. The efficient outcome is a level of flour use by firm #1 so that the marginal benefit to firm #1 — the value of the output produced by a marginal increase in flour use — equals the marginal social cost — the cost per kilo of the flour plus the value of the externality, which is the reduced profit to firm #2 from a slight increase in firm #1's four use.

If firm #1 ignores firm #2, it ignores the harm done to firm #1, and uses too much flour. There are many remedies to get firm #1 to choose the efficient level of flour use. Among them.

1) levy a tax on firm #1's flour use, equal to the marginal damage : the reduction in firm #2's profits from a slight increase in firm #1's flour use

2) subsidize firm #1's **reduction** in flour use (below the level it would choose if it ignored the externality), with the subsidy per unit reduction equal to the reduction in firm #2's profits from a slight increase in firm #1's flour use

3) simply order firm #1 to use no more flour than the efficient level

4) let firm #1 and firm #2 merge, so that the owner of the merged firm will try to maximize the combined profits of the two firms

5) do nothing, so that the owner of firm #2 will have to — and will choose to — bribe firm #1 to reduce its flour use to the efficient level

6) have a judicial decision that firm #1 does not have the right to use flour without firm #2's permission : then firm #1 will have to – and will choose to — bribe firm #2 for permission to use flour as an input, up to the efficient level