AP/ECON 4080 February 2014

Answers to Mid–term Exam

$Q1$. How would the addition of one more person into a country affect the well–being of the people already in the country, if the country provided a pure public good to its citizens, and used "Lindahl", or "benefit" taxation to pay for it?

[You may assume that each person in this country knows everyone else's preferences (including the new entrant's). Under Lindahl taxation, the quantity provided of a public good is the quantity for which the sum of people's marginal benefits of the public good equals the marginal cost of providing one more unit of the public good. And each person's tax bill is her own marginal benefit, times the quantity provided.]

$A1$. The short answer is that every person in the country must be made better off by the addition of one more person.

Let p_i be person i's Lindahl price, which equals her marginal benefit of the public good. Let Z be the quantity provided of the public good, and c the cost of providing one unit of the public good. Then at the Lindahl equilibrium, it must be true that

$$Z = D_i(p_i) \tag{1-1}$$

for each person i, where $D_i(p_i)$ is the person's demand curve for the pure public good. It also must be true that

$$p_1 + p_2 + \cdots + p_I = c \tag{1-2}$$

where I is the number of people.

Person i must be made better off if the quantity Z of the pure public good is increased by the addition of one more person. Why? Her demand curve slopes down. So her Lindahl price p_i will go down if and only if Z goes up. (That's what equation $(1-1)$ says.) And if p_i goes down, the person is better off : just as the fall in the price of an ordinary private good makes me better off if I consume the good, so must a fall in my Lindahl price, if I get to consume the quantity I demand, $D_i(p_i)$.

And adding one more person **must** increase the equilibrium quantity Z provided using the Lindahl mechanism defined by equations $(1-1)$ and $(1-2)$. Adding one more person means that the vertical sum of people's demand curves must shift out, raising the quantity Z.

[Alternately : if Z fell, then each person's p_i would have to **increase** since each person's demand curve slopes down. But if each existing person's p_i increased, and if we added one more person, then equation $(1-2)$ can't be satisfied any more : the left side would have increased with the right side staying the same.]

[Note : the result, that each existing resident is made better off by the addition of more people, depends on the assumption that Lindahl taxation is being used to finance the public sector. If some other tax were used, some people could be made worse off by the addition of new people to the country.]

*Q*2. How much tax revenue would be collected by the following "pivot tax" mechanism, if each person tries to use the mechanism to make herself as well off as possible?

The indivisible ("all or nothing") public project costs \$10000. There are 5 people : each person knows how much she values the project (but nobody else knows her valuation). Person #1 values the project at \$5000, person #2 and person #3 each value the project at \$1500, and person #4 and person #5 each value it at \$500.

The rules of the tax are : the project will be undertaken if and only if the average of people's announced valuations exceeds the cost per person of the project, \$2000. If the project is undertaken, each person will pay the same share, \$2000, of the cost. In addition, if any person is "pivotal" (that is, if her valuation alters the overall result), then she will have to pay a pivot tax, equal to the (absolute value of the) difference between the sum of everyone else's announced valuations and the sum of the shares of the cost (8000) which they must pay.

*A*2. A person will pay a pivot tax, on top of her share of the cost (if the project in undertaken) only if she is pivotal. She would be pivotal if her announced valuation switched the average of all announced valuations from less than \$2000 without her to more than \$2000 with her included, **or** if her announced valuation switched the average of all announced valuations from more than \$2000 without her to less than \$2000 with her included.

If people understand the mechanism, then each person will realize the best strategy is to reveal truthfully his or her valuation of the project.

If people announce their true valuations, the sum of their valuations is $5000+1500+1500+500+500=9000<10000$ and the project will **not** be built : the average of all 5 people's valuations is $1800<2000$.

Person 1 is **not** pivotal. The average valuation without her included is $\frac{1500+1500+500+500}{4}=1000<2000$. So with or without person 1, the project is rejected, which means that she is not pivotal.

Neither is person 2 pivotal. Without his valuation, the average valuation of the remaining 4 people is $\frac{5000+1500+500+500}{4} = 1875 < 2000$. So with or without person 2, the project is rejected, which means that he is not pivotal. The same must be true for person #3, since her valuation is the same as person #2's.

But person #4 **is** pivotal. The sum of the other 4 people's valuation, leaving out person #4, is $5000 + 1500 + 1500 + 500 = 8500$, so that the average of those 4 people's valuation **exceeds** the average cost per person of \$2000. Including person 4 switches the decision from "build" to "don't build". So he is liable for a pivot tax, equal to the difference between the sum of the other people's valuations, and their shares of the cost of the project : $8500 - 8000 = 500$.

Since person #5 has the same valuation as person #4, she too is pivotal, and is assessed a pivot tax of \$500.

Therefore, pivot tax revenue of \$1000 will be collected here, \$500 each from person #4 and person #5.

So if every person chose her or his best strategy, the outcome would be : the project is not undertaken ; no–one has to pay anything to cover the cost of the project ; people #4 and #5 each pay a pivot tax of \$500.

[Note that this extra pivot tax revenue will **not** be paid as compensation to people #1, #2, and #3.]

[Also note that, even if he could foresee the outcome, person #4 would not want to lie about his weak taste for the project. If he announced, for example, a valuation of \$2000, he would avoid paying any pivot tax. But that strategy would lead to him paying \$2000 in taxes for a project which he values at only \$500. Better to pay \$500 for nothing than \$2000 for something worth \$500.]

*Q*3. Suppose there is some input Z to production with the following properties : increases in firm 1's own purchases Z_1 of the input lead to increased profits for firm 2, and increases in firm 2's own purchases Z_2 of the input lead to increased profits for firm 1.

Is the equilibrium allocation efficient, when each firm chooses its own input quantities so as to maximize its own profit?

Explain briefly.

*A*3. In this example, there are positive externalities. If they ignore each other, each firm will hire **too little** labour, since they take account only of the impact of the labour they hire on their own output, and not on the output of the other firm.

If labour is in perfectly elastic supply at a cost of w per hour, and the level of firm 1's output can be written $Q_1 = F(L_1, L_2)$, then, ignoring the externality, firm 1 will hire labour up until the level L_1^0 at which

$$p_1 F_1 = w$$

where p_1 is the price of firm #1's output and F_1 is the derivative of F with respect to L_1. But firm 2's output is $G(L_1, L_2)$, depending on both firms' labour inputs, so that the efficient level of L_1^* is the level for which

$$p_1 F_1 + p_2 G_1 = w$$

Since $G_1 > 0$, $L_1^* > L_1^0$: firm 1 hires an inefficiently low quantity of labour, if it considers only its own private benefit of increased labour use, rather than the overall social benefit.

Similarly, firm 2 would choose, on its own, to hire the quantity L_2^0 of labour for which

$$p_2 G_2 = w$$

when the efficient level L_2^* of L_2 is the level for which

$$p_1 F_2 + p_2 G_2 = w$$

If firms cannot negotiate, or merge, government intervention may be required. The government could order both firms to increase their employment of labour (up to the efficient level at which the marginal increase from a firm's labour hiring on the sum of the value of both firms' outputs equals the wage per unit of labour). Or the government could subsidize both firms' use of labour : with the subsidy rate equaling the value of the marginal increase in the other firm's output.

However, if the firms merge, the new merged entity will take account of the externality from one branch of the new merged firm onto the other branch, and will hire the efficient quantity of labour for each branch.

Even if firms do not merge, if they can negotiate, they will be able to reach the efficient outcome without government intervention. Here the firms might agree to a deal in which firm 1 agrees to expand its labour hiring up to the overall efficient level, provided that firm 2 agrees to expand its labour hiring of to the overall efficient level.