AP/ECON 4080 February 2015

Answers to Mid–term Exam

*Q*1. What are the efficient allocations in the following 2–person economy, in which one of the two goods is a pure public good?

The marginal rate of transformation (MRT) between the two goods is a constant, equal to 1. The equation for the production possibility curve is $X + Z = 20$, where X and Z are the quantities produced of the two goods.

Person 1's preferences can be represented in the form

$$U^1(x_1, z_1) = x_1 + 2\log z_1$$

and person 2's in the form

$$U^2(x_2, z_2) = x_2 + 3\log z_2$$

where "log" refers to the natural logarithm function, x_i is person i's consumption of good X, and z_i is person i's consumption of good Z.

Good X is a **pure private good** and good Z is a **pure public good**.

*A*1. Any efficient allocation must satisfy the following conditions, if good X is a private good and Z a pure public good.

$$z_1 = z_2 = Z \tag{1-1}$$

(no–one should be excluded from the benefits of the public good)

$$x_1 + x_2 + Z = 20 \tag{1-2}$$

(the allocation should be on the production possibility curve)

$$MRS^1 + MRS^2 = MRT = 1 \tag{1-3}$$

(the Samuelson condition, where MRS^i is person i's marginal rate of substitution between the goods)

From the utility functions given in the question, and from the fact that the derivative of $\log z_i$ with respect to z_i is $1/z_i$, here

$$MRS^1 = \frac{2}{z_1} \tag{1-4}$$

$$MRS^2 = \frac{3}{z_2} \qquad (1-5)$$

so that the Samuelson condition $(1-3)$ becomes

$$\frac{2}{Z} + \frac{3}{Z} = 1 \qquad (1-6)$$

(where I have substituted $z_1 = z_2 = Z$ from condition $(1-1)$), or

$$Z = 5 \qquad (1-7)$$

In this case, there is a unique efficient level of public good provision, $Z = 5$. [Because the utility functions are quasi–linear, each person has an income elasticity of demand for the public good of 0, which means that each person's demand curve for the public good does not shift when her income changes.]

Since an efficient allocation must be feasible (equation $(1-2)$, the set of efficient allocations is the set of all (x_1, x_2, z_1, z_2) for which

(i) $z_1 = z_2 = 5$

(ii) $x_1 + x_2 = 15$

(iii) $x_1 \geq 0 \quad x_2 \geq 0$

$Q2$. What are the strengths and weaknesses of the "partial equilibrium" preference revelation mechanism for a town with I people, in which : (i) each person i announces a willingness–to–pay function $v_i(Z)$ as a function of the quantity Z provided of a public good ; (ii) the level provided of the public good is the Z^* satisfying $\sum_{i=1}^{I} v_i(Z^*) = MC$ where MC is the unit cost of the public good ; (iii) each person pays an equal share of the cost of the public good ; (iv) in addition, each person pays an extra tax equal to the area $\int_{Z_i}^{Z^*} [MC\frac{I-1}{I} - \sum_{j\neq i} v_j(Z)]dZ$, where Z_i is the level of public good which would be provided if person i were not included? [Z_i is defined by the equation $\sum_{j\neq i} v_j(Z_i) = MC\frac{I-1}{I}$.]

$A2$. This is the preference revelation mechanism described in part (b) of the section of "Preference Revelation".

The main strength of the mechanism is that each person, if she understands the rules, will choose to announce truthfully her willingness–to–pay function, however other people choose to answer. That is, telling the truth is a dominant strategy with this mechanism.

To see this, let $b_i(Z)$ denote the **true** marginal willingness–to–pay function of person i. And let T_i be person i's taxes. Parts (iii) and (iv) of the rules imply that person i's tax bill depends only on the quantity Z provided of the public good (and on other people's announced marginal–willingnesses–to–pay, over which person i has no influence).

Person i would like her total benefit $B_i(Z)$ from the public good, minus her tax bill $T_i(Z)$, to be as large as possible. So to maximize these net benefits, she would like to find the level Z of public good provision for which

$$b_i(Z) - \frac{\partial T_i}{\partial Z} = 0 \tag{2-1}$$

Since

$$T_i(Z) = \frac{MC}{I}Z + \int_{Z_i}^{Z^*} [MC\frac{I-1}{I} - \sum_{j \neq i} v_j(Z)]dZ \tag{2-2}$$

from parts (iii) and (iv) of the rules, therefore

$$\frac{\partial T_i}{\partial Z} = \frac{MC}{I} + MC\frac{I-1}{I} - \sum_{j \neq i} v_j(Z) \tag{2-3}$$

which means that

$$\frac{\partial T_i}{\partial Z} = MC - \sum_{j \neq i} v_j(Z) \tag{2-4}$$

Therefore (from equation $(2-1)$), the level of public good spending which is best for person i, given everyone else's announced marginal–willingnesses–to–pay, is the Z for which

$$b_i(Z) = MC - \sum_{j \neq i} v_j(Z) \tag{2-5}$$

or

$$b_i(Z) + \sum_{j \neq i} v_j(Z) = MC \tag{2-6}$$

The rule (ii) for the actual choice of the quantity Z of the public good can be written

$$v_i(Z) + \sum_{j \neq i} v_j(Z) = MC \tag{2-7}$$

Comparing equations $(2-6)$ and $(2-7)$, person i can get the best possible level of public provision by setting $v_i(Z) \equiv b_i(Z)$, that is by reporting truthfully her marginal–willingness–to–pay function $b_i(Z)$.

The "extra tax" defined in part (iv) of the rule must be, by definition, non–negative, and will be positive whenever $Z \neq Z_i$. So the rule guarantees that tax revenues must be at least as large as the cost of the public good, which is $(MC)(Z)$ (paid for by the tax in part (iii) of the rule).

However, one of the weaknesses of the mechanism is this budgetary surplus. The revenue collected through part (iv) of the rules **cannot** be given back to the town's residents, and cannot be spent on projects useful to the townspeople. Otherwise, these people would take into account the benefit from this extra tax revenue, and try to answer so as to increase this revenue, distorting their incentives to tell the truth.

There are two other main drawbacks. Since this is a partial equilibrium mechanism, in which people have to announce their demand curve for the public good without knowing what their after–tax income will be, it can only be used if the income elasticity of people's demands for the public good is zero. As well, although telling the truth is a dominant strategy for each individual, the mechanism can be exploited by collusion among people.

$Q3$. Suppose that the oil from an oil storage facility leaks into the ground, damaging the crops at a nearby farm.

Should the government : (i) shut down the storage facility ; (ii) tax the storage facility, based on the quantity of oil stored there ; (iii) place a limit on the quantity of oil which the facility can store ; (iv) do nothing?

Explain briefly.

$A3$. Assume, as stated in the question, that there are only 2 parties affected by this externality : the 1 oil storage facility and the 1 nearby farm. Then the appropriate remedy depends on the possibility of negotiation between the storage facility's owner's and the farm's.

If the oil storage industry and the farming industry are perfectly competitive, then the efficient outcome is that which maximizes the sum of the profits of the storage facility and the farm.

If the two parties can negotiate, then they will reach an agreement, specifying how much oil can be stored in the storage facility. This negotiated quantity will be the efficient level (at which the marginal benefit to the facility of storing a little more oil exactly equals the marginal damage done to the farm by this slight increase in the quantity of oil). If the government does nothing, it will be in the interest of the farm's owner to pay the storage facility's owner to reduce its quantity of oil stored to the efficient level. If the government places a limit on the quantity of oil (remedy # (iii)), and if this limit can be changed with the agreement of both parties, then, again, both parties will negotiate a change in the limit : the farm's owner will pay for a further reduction if the government–chosen limit is above the efficient level, and the storage facility's owner will pay the farm's owner to agree to an increase in the limit, if the original government–chosen limit is below the efficient level.

If the 2 parties can negotiate, then a tax (remedy #(ii)) may not lead to an efficient outcome : since the tax revenue is going to the government, then the sum of the two firms' profits — net of taxes — will be maximized at a level of oil storage which is **less** than the efficient level.

If the 2 parties cannot negotiate, then the government can achieve the efficient outcome by remedy #(iii), if it has enough information (about marginal costs and benefits of oil storage) to calculate the efficient level of oil storage. It can also achieve the efficient outcome with taxation, remedy #(ii), if it knows the marginal damages to the farm of oil storage : the correct tax rate, per barrel of oil, is the marginal damage from storage of one more barrel (evaluated at the optimum).

Shutting down the oil storage facility (remedy #(i)) will only be efficient in either of the following two cases. 1. The firms can negotiate, and the shutdown order can be cancelled with the agreement of the farm's owners. 2. The firms cannot negotiate, the government, for some reason, has to choose between the two extreme options of shutting down the storage facility and doing nothing, the first of those two options leads to a higher total profit for the two firms.