AP/ECON 4080 February 2016

Answers to Mid–term Exam

$Q1$. What are the conditions which must be satisfied for an allocation to be efficient in an economy in which there are many people, and $m + n$ goods : m private goods and n public goods?

$A1$. There are three types of efficiency condition for this economy :

- The aggregate production plan must be feasible, and on the production possibility frontier.
- Each person's marginal rate of substitution between any two private goods must be the same, and must equal the marginal rate of transformation between those goods.
- The sum of all the people's marginal rates of substitution between any public good and any one of the private goods, must equal the marginal rate of transformation between the public good and that private good.

Mathematically, if X_j represents the aggregate quantities of the two private good j, and Z_k represents the aggregate quantity of public good k, and if x_j^p and z_k^p represent person p's consumption of private good j and of public good k, then we must have

- $X_j = x_j^1 + x_j^2 + \cdots + x_j^P$ for each private good j if there are P people in the economy
- $z_k^1 = z_k^2 = \cdots = z_k^P = Z_k$ for each public good k
- the aggregate production $(X_1, X_2, \ldots, X_m, Z_1, Z_2, \ldots, Z_n)$ must be on the production possibility frontier
- if the preferences of person i can represented by a utility function $U^i(x_1^p, x_2^p, \ldots, x_m^p, z_1^p, z_2^p, \ldots, z_n^p)$, then

$$MRS_{j1}^1 = MRS_{j1}^2 = \cdots = MRS_{j1}^P = MRT_{i1}$$

where MRS_{j1}^p is person p's marginal rate of substitution between private good j and private good 1 [the amount of private good 1 she is willing to give up to get a little more of private good j], and MRT_{j1} is the marginal rate of transformation between private goods j and 1

- and for any public good k, and any private good j, it must be true that

$$MRS^1_{kj} + MRS^2_{kj} + \cdots MRS^P_{kl} = MRT_{kj}$$

where MRS^p_{kj} is person p's marginal rate of substitution between public good k and private good j, and MRT_{kj} is the marginal rate of transformation between goods k and j

$Q2$. How much tax revenue would be collected by the following "pivot tax" mechanism, if each person tries to use the mechanism to make herself as well off as possible?

The indivisible ("all or nothing") public project costs \$6000. There are 6 people : each person knows how much she values the project (but nobody else knows her valuation). Person #1 and person #2 each value the project at \$2000, person #3 and person #4 each value the project at \$1500, and person #5 and person #6 each value it at \$500.

The rules of the tax are : the project will be undertaken if and only if the average of people's announced valuations exceeds the cost per person of the project, \$1000. If the project is undertaken, each person will pay the same share, \$1000, of the cost. In addition, if any person is "pivotal" (that is, if her valuation alters the overall result), then she will have to pay a pivot tax, equal to the (absolute value of the) difference between the sum of everyone else's announced valuations and the sum of the shares of the cost (5000) which they must pay.

$A2$. The project will be undertaken if the average of the six people's announced valuations v_i exceed the cost per person of the project, \$1000.

A person will pay a pivot tax, on top of her share of the cost (if the project in undertaken) only if she is pivotal. She would be pivotal if her announced valuation switched the average of all announced valuations from less than \$1000 without her to more than \$1000 with her included, or vice versa.

If people understand the mechanism, then each person will realize the best strategy is to reveal truthfully his or her valuation of the project.

If people announce their true valuations, the sum of their valuations is $2(2000 + 1500 + 500) = 8000$ and the project will be built : the average of all 6 people's valuations is $1333.33 > 1000$.

So a person will be pivotal only if the average of **everyone elses'** valuations is less than \$1000.

If we leave out person #1, then the average of the remaining 5 people's valuations is $\frac{2000+1500+1500+500+500}{5} = 1200 > 1000$. Person #1 is not pivotal if everyone reveals their

valuations truthfully : even without her, the average of the valuations exceeds the cost per person of the project.

Since person #2 has an identical valuation to that of person #1, then he will not be pivotal : the average of the valuations of people $1, 3, 4, 5$ and 6 is also 1200.

Since people $3, 4, 5$ and 6 have lower valuations than people 1 and 2, the average valuation without any one of them is certainly more than 1000. [It's actually 1300 if person 3 or 4 is left out, and 1500 if person 5 or 6 is left out.]

So, if people understand the mechanism and realize that they are best off reporting their true valuations, then no–one will have to pay any pivot tax in equilibrium.

$Q3$. If fruit production in some farm was an increasing function of the number B of bees used at another, nearby firm which uses the bees as an input to honey production, describe two different policies which ensure that the honey firm chooses an efficient quantity of bees to use.

$A3$. The efficient outcome is a quantity B of bees used by the honey farm such that the marginal benefit to the honey farm — the value of the increased honey production from a marginal increase in the number of bees — plus the marginal benefit to the fruit farm — the value of the added fruit production due to the increase in B equals the marginal cost of the bees.

If the honey firm ignores the effects of its bees on fruit production, it ignores some of the marginal social benefits of increased bee use, and uses too few bees. There are many remedies to get the honey firm to choose the efficient number of bees. Among them

1) subsidize the honey firm's use of bees ; the subsidy per bee should equal the value of the marginal increase in fruit production due per extra bee, evaluated at the efficient outcome

2) **tax** any **reduction** in use of bees by the honey farm (below the efficient level), with the tax per unit reduction equal to the reduction in the fruit farm's profits from a slight decrease in bee use by the honey farm

3) simply order the honey firm to increase B to the efficient level

4) let the fruit farm and the honey firm merge, so that the owner of the merged firm will try to maximize the combined profits of the two firms

5) do nothing, so that the owner of the fruit farm will have to — and will choose to —

bribe the honey firm to increase B to the efficient level

6) have a judicial decision that the honey firm must employ some very large quantity of bees, unless the fruit farm agrees to let the honey firm reduce B ; in this case the honey firm's owner will bribe the fruit farm to let it reduce B, if the judge set the quantity B above the efficient level

Notice that, if negotiation is costless, some of these remedies do not require government involvement.