Econ 4080.03MW Midterm Answers
March 1997

1. When there is only one private good, only one public good, and many people, then optimality requires the Samuelson condition

$$\sum_h MRS_h = MRT$$

to hold.

Given that one good is a pure public good, optimality also requires that each person consume all the available public good :

$$g_1 = g_2 = \cdots = g_H = G$$

The only other efficiency condition is that the economy be efficient in production, that is that the production plan be on the production possibility curve, which here is a straight line with slope of $-1$.

So the only remaining step to answer the question is to figure out the marginal rate of substitution when each person's preferences can be represented in the form

$$U(x_h, g_h) = x_h + a_h\sqrt{g_h}$$

The marginal rate of substitution ( as recalled from Econ 1000 and Econ 2300 ) is the ratio of the marginal utilities. Here the marginal utility of the private good – the derivative of the utility function with respect to $x_h$ — is just 1. The marginal utility of public good consumption is the derivative of the utility function with respect to $g_h$, which is $(1/2)a_h/\sqrt{g_h}$. So

$$MRS_h = \frac{1}{2}\frac{a_h}{\sqrt{g_h}}$$

and the efficiency consition is

$$\sum_h \frac{1}{2}\frac{a_h}{\sqrt{G}} = 1$$

or

$$G = \frac{1}{4}[\sum_h a_h]^2$$

[ A large number of answers did not try to answer the question. The question did not ask for a derivation of the Samuelson condition $\sum MRS_h = MRT$. It asked to apply them in the particular example. ]

2. Replacing simple majority rule with the requirement that the new policy proposed get two–thirds of the votes in order to supplant the status quo will NOT get rid of cycling, as the following example shows.

Suppose that person one prefers policy $A$ to policy $B$ to policy $C$ ; person 2 prefers policy $B$ to policy $C$ to policy $A$, and person 3 prefers policy $C$ to policy $A$ to policy $B$. ( This is the example I used to show the existence of cycling under simple majority rule. )

Here a simple majority preferring one alternative to another is *exactly the same* as two–thirds of the voters preferring that alternative. In either case, two of the three people preferring the alternative is what is required.

[ Note that the Arrow impossibility theorem does not say that cycling is inevitable. It says that any decision procedure which eliminates cycles will have other problems, for example, dependence on irrelevant alternatives when ranking two policies. ]

[ Note also that the median voter theorem is not really relevant here. The median voter theorem applies to one particular decision procedure, voting over pairs of alternatives, with a fifty percent majority required for the new proposal to defeat the status quo. And if the median voter theorem applies, cycling will not be a problem. ]

[ Note also that the "64% majority" paper by Caplin and Nalebuff ( mentioned in Stiglitz ) does not apply to the three–person, three–policy example described above. It applies only when people have single–peaked preferences over multi–dimensional alternatives. ]

3. If a good is produced under increasing returns to scale, then the industry that produces the good *may* be a natural monopoly. A natural monopoly is an industry in which the monopolist can charge its profit–maximizing price without worrying about the threat of entry by any other firm.

So an industry can have increasing returns and still not be a natural monopoly.

But even if the industry is not a natural monopoly, it still may be a monopoly — with the monopoly choosing a price which deters entry by any other firm.

An industry with increasing returns, and no sunk costs, is what has been called a "contestable" industry. Such an industry is *not* a natural monopoly, but it will be a monopoly. Here the monopoly's equilibrium price is set so that the firm breaks even.

Under increasing returns, a ( single–product ) firm's average cost decreases with the output produced. Therefore the average cost exceeds the marginal cost. The efficient level of output is always the level of output such that the price buyers are willing to pay equals the marginal cost. Under increasing returns, then, a firm producing the efficient level of output will lose money, since average cost exceeds marginal cost which equals the price charged.

An ordinary single–price monopoly chooses the output level for which marginal cost equals marginal revenue. This output level is inefficiently low. It also is lower than the level of output which causes the firm to break even ( assuming that the demand curve and average cost curve do cross ).

So natural monopoly leads to a big deadweight loss, compared with the efficient level of output. The break–even quantity leads to a smaller deadweight loss.

If government provision of the good involves exactly the same cost function as private provision, then the government could produce the efficient level of the good, and cover the resulting loss with tax revenue. Collecting that tax revenue may involve a deadweight loss of its own. But government provision will be at least as efficient as unregulated private provision.

Of course, there is some evidence that the cost of producing a given level of output will be higher under public provision than under public provision. The explanation for these higher costs may lie in the weaker incentives for cost reduction in a public enterprise, which does not face competition or the threat of bankruptcy. But remember, a natural monopoly does not face these threats either.

[ The question referred to government provision, rather than government regulation. So the information problems faced by the government trying to regulate a private firm with private information about its cost structure are not relevant to the question. ]