AP/ECON 4380 & GS/ECON 5950
(Brief) Answers to Final Exam
June 2011

1. Suppose that there are 5 voters of type #1, 6 voters of type #2, 7 voters of type #3, 8 voters of type #4 and 9 voters of type #5, with the following preference orderings over candidates

	[5 people]	[6 people]	[7 people]	[8 people]	[9 people]
first choice	v	z	y	z	x
second choice	w	w	z	v	w
third choice	x	x	v	w	y
fourth choice	y	v	w	x	z
fifth choice	z	y	x	y	v

Is there a Condorcet winner here? Explain briefly.

answer : There is **no** Condorcet winner here. To prove this, I must show that each of the 5 candidates will be defeated, in a pairwise contest, but at least one other candidate.

Note that here what it takes to win a two–candidate race is to be preferred by at least 3 types (any 3). Even with the least numerous types, #'s 1,2 and 3, there are 18 voters of type #1, 2 or 3, and only 17 of types #4 and 5.

So candidate v is preferred to candidates w, x and y by at least 3 types of voter. So none of candidates w, x and y can be a Condorcet winner ; they all lose to v. That leaves v and z as possible Condorcet winners. Candidate z is preferred to candidate v by voters of type #2,3,4 and 5, and so v loses a two–way election to z.

That leaves z as the only possible Condorcet winner.

However, voters of type #1, 3 and 5 all rank y above z, so that z loses to y in a two–candidate election, which means that z is **not** a Condorcet winner either.

2. Which of the axioms of Arrow's Impossibility Theorem does the following choice rule violate?

Take all the different **pairs** of alternatives, and number them. (So if there were 4 alternatives a, b, c and d, then there would be 6 pairs of alternatives : a versus b, a versus c, a versus d, b versus c, b versus d and c versus d).

Then let person 1 make the choice for **pair** #1 (so that if pair #1 is alternative a versus alternative b, we would rank a above b if and only if person #1 ranked a above b). Let person #2 make the choice for pair #2, and so on. If there are more pairs of alternatives than people, then keep going back around to person #1 after we get to the last person. (So with 8 people, and 10 pairs of alternatives, person #1's preferences would determine the ranking of pairs #1 and #9, person #2's preferences would determine the ranking of pair #2 and #10, person #3's preferences would determine the ranking of pair #3 and so on.)

answer : The one axiom which this choice rule violates is **transitivity**.

Two ways to show this. One is direct : Suppose that person #1 ranked a above b, person #2 ranked b above c and person #3 ranked c above a. Then if a versus b is pair #1, b versus c is pair #2, and a versus c is pair #3, then the rule generates a cycle, a preferred to b preferred to c preferred to a, violating transitivity.

The indirect way is to show that the rule obeys all the other 4 axioms. It does. First, there is "universal domain" ; the rule here can be applied for any number of people, and number of alternatives, and any preference profiles for the people.

Next is "non–dictatorship". No person here is a dictator over **all** choices. Person #1 gets to be a dictator for pair #1, but not for pair #2. And the statement of this axiom (pg. 583 of *Mueller*, for example) says that a person's choices must dominate for "**any** two alternatives" for the person to be a dictator.

The "Pareto" principle holds here : if every person ranks a above b, then in particular the person who gets to make that choice over that pair ranks a over b, so that the rule must rank a over b if everyone prefers a to b.

Finally there is the "independence of irrelevant alternatives". That says that, for any pair of alternatives, if no person changes the way she ranks those two alternatives, the rule won't change. If no person changes the way she ranks the two alternatives in pair #3, for example, then, in particular, person #3 hasn't changed the way she ranks them. That means the rule (which takes into account only person #3's ranking) won't change the way the alternatives are ranked.

So the rule obeys all 4 axioms, other than transitivity. Since Arrow's Impossibility Theorem says that every choice rule must violate at least one of the axioms, this rule must violate transitivity.

3. What are the key assumptions needed to get the result that all parties will choose the median of the voters' most–preferred policies?

   answer This is a question about the "Hotelling–Downs" model of political parties, in which the parties' platforms are virtually identical in equilibrium.

   The main assumptions needed to get this result are

(a) There are only two parties.

(b) Parties can choose their platforms freely, and can commit to a platform.

(c) Platforms can be represented as points on a line.

(d) Voters have single–peaked preferences over these platforms.

(e) Parties care only about maximizing the number of votes that they get.

(f) Voters always vote, and always vote for the party platform which they prefer.

The equilibrium result is that parties will both choose a platform which is the median of the voters' preferred platforms, if they make their platform choices non–cooperatively and simultaneously.

Assumptions (3e) and (3f) actually may not be crucial ; the principle of minimum differentiation may hold even if parties have their own ideology, and even if voters' decision whether to participate depends on the party platforms. But if there were more than 2 parties, or if platforms were multi–dimensional, then this principle of minimum differentiation will not occur.

4. What level of service provision X and what budget B should a government official propose if she wants the largest budget B possible, in the following circumstances? The budget B she requests must cover the cost $C(X)$ of providing the service where

$$C(X) = 2X^2$$

She can propose any service level X and budget B which she wishes (provided that $B \geq C(X)$). However, the proposal must pass a referendum of the voters.

If the proposal does not pass, the budget will be 0, and the level of service provision will be 0.

Each voter gets a total benefit of

$$W(X) = 30X - X^2$$

from a public service level X, and cares only about his total benefits $W(X)$, minus the budget B which is financed from his taxes.

answer : This model is the Niskanen model of bureaucracy.

The government official's proposed service level X and her proposed budget B have two constraints that must be satisfied. The budget must cover the cost of services :

$$B \geq C(X)$$

And the service–budget combination must be approved by the voters. This second constraint can be written

$$W(X) - B \geq 0$$

because the service–budget combination must be perceived as better than the alternative, which here is $X = B = 0$.

The first constraint must bind. If the government official asked for more budget than necessary ($B > C(X)$), and still got the service–budget package approved, then she could increase the budget even further, by increasing both X and B slightly, so that $W(X)$ increased by the same amount as B. As long as $B > C(X)$ these increases would keep her within budget ; by keeping $W(X) - B$ unchanged, she ensures the new package will still be approved. So if $B > C(X)$ the government official is not maximizing her budget ; another proposal can yield her a bigger budget, and still get passed.

Since the first constraint must bind, we can set $B = C(X)$, so that the second constraint can be written

$$W(X) - C(X) \geq 0$$

Since $B = C(X)$ and $C'(X) > 0$, the government official wants X as large as possible, given the above constraint.

Given the functional forms given in the question, we are looking for the largest X such that

$$F(X) \equiv W(X) - C(X) = 30X - X^2 - 2X^2 = 30X - 3X^2 \geq 0$$

Since $F(0) = 0$, and $F'(X)$ for small X and $F''(X) = -6 < 0$, the function $F(X)$ increases at first as X rises above 0, and then decreases. It reaches 0 again at the largest value of X such that

$$30X - 3X^2 = X(30 - 3X) = 0$$

or $X = 10$

So the government official provides a service level of $X = 10$, asks for a budget of $B = C(10) = 200$, and gets the proposal (barely) approved, since $W(10) = 200 = B = C(10)$. This level is much higher than the service level which maximizes the net welfare of the voters. That level of service, for which $F'(X) = 0$ is $X = 5$

5. How much infrastructure spending will be spent in district 1 of a legislature in the following situation? Explain briefly.

   There is a fixed budget of \$1 billion for infrastructure spending, which must be divided up among geographically concentrated projects. Each project is located in a single district, and there are many competing projects.

   There are 101 districts represented in the legislature. The representative from district 1 has been chosen to propose a bill dividing up the budget. But any bill she proposes must be passed by a simple majority of the legislature. If it is defeated, someone else will be chosen (randomly) to propose a new bill.

   answer : This is basically Harrington's model of the role and power of an agenda setter over distributive spending, discussed in section 5.12.2 of *Mueller*.

   The agenda setter needs the support of 50 other legislators in order to pass her bill. So she must provide some spending in the districts of these 50 legislators.

   How much spending does it take to win the support of these other legislators? Let x be the amount of spending offered by the agenda setter to a potential supporter. That means a total of $50x$ in the districts of the 50 legislators whose support the agenda setter is seeking. That leaves $1 - 50x$ billion dollars for the agenda setter — since she has no reason to seek the support of more than 50 legislators.

So what happens if the proposed bill fails? There will be another bill. The other bill will propose spending x in each of 50 districts, 0 in 50 other districts, and $1 - 50x$ in the home district of the next agenda setter. Each legislator has a 1/101 chance of being the next agenda setter, a 50/101 chance of being in the next period's winning coalition, and a 50/101 chance of being left out next period. So a typical legislator will expect to get spending of

$$\frac{1}{101}(1 - 50x) + \frac{50}{101}x + \frac{50}{101}0$$

if the current bill is defeated, and next period a new one is proposed.

The smallest value of x which will induce a legislator to vote for the **current** bill is the x which gives that legislator the same level of spending in his district as he would get, on average, if the current bill were defeated, so that

$$x = \frac{1}{101}(1 - 50x) + \frac{50}{101}x + \frac{50}{101}0 \qquad (*)$$

Solving $(*)$ for x,

$$x = \frac{1}{101}$$

So the current agenda setter, the representative for district 1, should offer the smallest possible amount of spending, in the smallest possible number of districts, that it takes to get her bill passed. She should offer $\frac{1}{101}$ billion dollars of spending in each of 50 other districts, leaving the remaining $\frac{51}{101}$ billion dollars (just over \$500,000,000) to be spent in her own district 1.

6. What determines whether some category of public expenditure is best provided at the national, provincial or local level?

   answer The traditional literature on multi–level government emphasizes two characteristics of the different categories of public expenditure.

   The first is the technology : how the cost of the expenditure varies with the population being provided. If economies of scale in population are important, that suggests that a "higher" level of government (such as the the national government) should be responsible for the category. (Economies of scale in population mean that the cost per person falls as the population increases.)

   The second feature is diversity of taste. If people differ in their demands for the public expenditure, this diversity suggests that lower levels of government (such as local) should be responsible. This argument is based on the notions that people are mobile, and can sort themselves (or have sorted themselves) according to their tastes for the public expenditure category. Implicit in the argument is the assumption that a jurisdiction, whether local, provincial or national, is somehow constrained to provide a uniform level of public service everywhere.

   More recent analysis of multi–level government recognizes that uniformity of provision may not be a physical constraint imposed on a government. But most democratic governments have legislatures in which different geographic regions have different representatives. Having a high level of government decide on "distributive" public expenditure may therefore be inefficient, as log–rolling by legislators results in too much spending in some districts (those represented by members of the winning coalition) and too little spending in other (those represented by members outside the winning coalition).

Another argument against provision by higher levels of government is information asymmetry. Local politicians may have better knowledge of tastes for public expenditure, or of the cost of public expenditure provision, in their own jurisdiction.

Another argument for provision at a higher level of government is the presence of spillovers among jurisdictions. If public expenditure in one city benefits residents of other cities, then self–interested local governments will spend to little on this category if local governments make their decisions non–cooperatively. Provision by a higher level of government, or a system of conditional matching grants from a higher level of government will lead to a higher, more efficient, level of provision.

Competition for tax base among jurisdictions is an argument for leaving more taxing authority to higher levels of government, although it does not directly affect the allocation of expenditure responsibility among levels of government. However, this tax competition may result in lower levels of government spending too much on some types of public infrastructure, in an attempt to increase their tax bases by attracting businesses or high–income people.

7. Suppose that the income elasticity of demand for highway expenditure is 1, and that highway expenditure in New Brunswick equals 1% of total provincial income.

   Theoretically, what would be the effect on New Brunswick's highway expenditure of a categorical non–matching grant of $100,000,000 from the federal government to the New Brunswick government, which must be spent on highway construction and maintenance?

   How does the "flypaper effect" modify this theoretical prediction?

   answer If the New Brunswick government acts like a utility–maximizing consumer, then theory predicts that a $100,000,000 conditional non–matching grant, which must be spent on highway construction and maintenance, should have exactly the same effect as an increase in total income of $100,000,000 for the population of New Brunswick.

   So what would be the effect on highway expenditure in New Brunswick of an increase in New Brunswick's income of $100,000,000? If the income elasticity of demand for highway expenditure is 1, that says that highway expenditure should increase by the same percentage as income did.

   In particular

$$\frac{\partial H}{\partial Y} = \eta \frac{H}{Y}$$

   where η is the income elasticity of highway expenditure demand, H is total highway expenditure and Y is total income in New Brunswick. If highway expenditure in New Brunswick equals 1% of total income, that says that $\frac{H}{Y} = 0.01$ so that each dollar increase in New

Brunswick's income leads to a 1–cent increase in highway expenditure. That means a \$100,000,00 grant should lead to an increase in highway expenditure of \$1,000,000. That's what theory predicts would be the effect of a \$100,000,000 non–matching grant.

There are two theoretical qualifications. First, the grant from the federal government is paid for by taxes on Canadians. Some of those Canadians live in New Brunswick. Since New Brunswick has about 2% of Canada's population, its residents are paying about 2% of the cost of any federal expenditure. So New Brunswick residents should realize that a grant of \$100,000,000 really represents a new increase in income of a little less than that, perhaps about \$98,000,000, since about \$2,000,000 of the cost of the grant is being paid by the province's own residents.

Second, the way in which the province can treat a categorical non–matching grant just like a general income increase, is by taking advantage of the fungibility of spending. It has to spend the grant on highways, but it can compensate by lowering spending on highways from its own tax revenues. In this case, since it wants an overall increase in highway expenditure of \$1,000,000, it should reduce by \$99,000,000 spending on highways from its own tax revenues. The complication is that it may not have that much to reduce. What if overall spending on highways in New Brunswick — before the grant — was less than \$99,000,000? Suppose, for example, it were only \$70,000,000. Then the "best" the provincial government can do is to eliminate totally its own spending in highways, leaving total expenditures on highways at \$100,00,000 entirely funded by the grant. That's an increase in highway expenditure of \$30,000,000, not \$1,000,000.

[The actual budget of the New Brunswick Ministry of Transportation for 2010–11 was about \$185,000,000.]

The "flypaper effect" is a label for an empirical regularity : the effects of categorical, non–matching grants do not follow the predictions of theory. Usually, the overall increase in spending on a category, following a grant of this type, appears to be close to the actual amount of the grant. That is, theory predicts the grant should increase overall highway expenditure by \$1,000,000 ; experience suggests the grant would increase highway expenditure by something close to \$100,000,000.

8. What (if anything) is the net gain to the cable television industry, if the government of some country decided to award a monopoly franchise worth \$400 million in profits in some metropolitan area when there are four firms in the industry, and each firm can spend money L_i in lobbying the government, and the probability that firm i is awarded the franchise is

$$\pi_i = \frac{\sqrt{L_i}}{\sum_{j=1}^{4} \sqrt{L_j}}$$

when firms choose non–cooperatively how much to spend on lobbying activities?

answer : Formula (15.4) of *Mueller* applies here, with $R = 400, n = 4, r = 0.5$, so that each firm in equilibrium will spend

$$L_i = \frac{n-1}{n^2} rR = \frac{3}{16}(0.5)400 = \frac{300}{8}$$

(in millions of dollars), and all 4 firms together spend

$$nL_i = 4\frac{300}{8} = 150$$

So firms spend 150 million dollars in lobbying,meaning the net gain to the industry is 250 millions dollars, the value of the franchise minus the amount the firms spent in lobbying against each other to get the franchise.

To derive this result without using Mueller's formula, note that each firm chooses its own lobbying expenditure L_i so as to maximize its expected return, which is

$$400\pi_i - L_i$$

The first term above in the expression above is the expected profits from the cable television franchise, and the second is the cost of lobbying. Given the expression in the question for π_i, firm 1, for example, chooses L_1 so as to maximize

$$400\frac{\sqrt{L_1}}{\sqrt{L_1}+\sqrt{L_2}+\sqrt{L_3}+\sqrt{L_4}} - L_1$$

Differentiating this expression with respect to L_1 (noting that L_1 appears in both numerator and denominator of the first term), and setting the result equal to 0,

$$400[\frac{1}{2\sqrt{L_1}}][\frac{1}{\sum_{j=1}^{4}\sqrt{L_j}} - \frac{\sqrt{L_1}}{[\sum_{j=1}^{4}\sqrt{L_j}]^2}] - 1 = 0$$

which can be written

$$200[1-\pi_1] = \sqrt{L_1}[\sum_{j=1}^{4}\sqrt{L_j}]$$

In a symmetric Nash equilibrium, all 4 firms make the same decision, so that $L_1 = L_2 = L_3 = L_4 \equiv L$, $\pi_1 = \frac{1}{4}$ and the above equation becomes

$$\frac{3}{4}200 = 4L$$

or

$$L = 37.5$$

In equilibrium each of the four firms spends 37.5 million dollars on lobbying, meaning the 4 firms are spending 150 million in aggregate, as the Mueller formula indicated.