AP/ECON 4380 & GS/ECON 5950
(Brief) Answers to Final Exam
April 2014

1. Give an example of a profile of voters, with preferences over several alternatives, which has the following property.

   Some alternative (say x) is a Condorcet winner, **but** this alternative x is the first alternative eliminated if Coombs's voting rule is used : under Coombs's rule, a sequence of ballots is held ; the alternative which is ranked last by the most voters is eliminated after the first ballot ; then the process is repeated (with this one alternative eliminated from the ballot), and then the alternative with the most last–place votes in this second ballot is also eliminated ; and so on, until only one alternative remains.

   answer The following table, in which there are 5 voters and 5 alternatives, illustrates one example :

	voter #1	voter#2	voter #3	voter # 4	voter #5
first choice	y	z	y	z	y
second choice	w	w	z	v	w
third choice	x	x	v	w	x
fourth choice	v	v	w	x	z
fifth choice	z	y	x	y	v

   Since alternative y is the first choice overall of three of the five voters, it will win any pairwise contest with any other alternative : voters #1, 3 and 5 prefer y to any other alternative.

   But 2 of the voters rank y last, and none of the other alternatives is ranked last by any of the other voters, so that y would be the first alternative eliminated under Coombs's rule.

   (In this example, z would be the winner under Coombs's rule.)

2. Prove the following result, which is used in the proof of Arrow's Impossibility Theorem :

   If some individual is decisive over some pair of alternatives, then that person is decisive over all pairs of alternatives, if the rule for the social ordering obeys the axioms of unrestricted domain, the Pareto principle (if everyone ranks x above y, then the ordering ranks x above y), and the independence of irrelevant alternatives.

[An individual is decisive over the pair of alternatives x and y if the social ordering ranks x above y whenever : that person ranks x above y and everybody else ranks y above x.]

answer Suppose that person 1 is decisive for x over y .

Suppose next that the profile of voters rankings of the three alternatives x , y and z is as depicted in the following table :

	voter #1	everyone else
first choice	x	y
second choice	y	z
third choice	z	x

Since voter #1 is decisive for x over y , the social ordering must rank x above y . By the Pareto principle, the social ordering must rank y above z , since all voters prefer y to z . Since a social ordering is transitive, then the social ordering for the profile above must therefore rank x above z . This is true even though every voter except for voter #1 prefers z to x .

Now consider any other profile of voter's rankings, in which voter #1 prefers x to z , and in which all the other voters prefer z to x . The axiom of the independence of irrelevant alternatives implies that the ranking of x versus z for this new social ordering must be the same as for the profile listed at the top : no–one has changed how they feel about x versus z , so that the social ordering of x versus z cannot change if the social ordering obeys the axiom of the independence of irrelevant alternatives.

So : if voter #1 is decisive for x over y , then she will be decisive for x over any other alternative.

The same sort of argument can be used to show that voter #1 will also be decisive for w over z , for any other alternatives w and z . Start with the profile :

	voter #1	everyone else
first choice	z	y
second choice	x	w
third choice	y	z
fourth choice :	w	x

If voter #1 is decisive for x over y , then the social ordering must rank x above y . If the social ordering obeys the Pareto principle, it must rank y above w , and z over x , since all voters prefer y to w and z to x . Since the social ordering is transitive, therefore, it must order the alternatives : z above x above y above w .

So the social ordering ranks z above w . The axiom of the independence of irrelevant alternatives states that the social ordering then must rank z above w for **any** profile in which voter #1 ranks z above w , and in which all the other voters rank w above z .

But this second example then shows : if voter #1 is decisive for x over y , she will be decisive over any other pair of alternatives.

3. Describe an equilibrium in which exactly 2 candidates choose to run in the "citizen–candidate" model, in which : candidates' own preferred policies are known ; candidates care about which policy is chosen ; candidates cannot commit to a policy other than their own preferred policy ; it is costly for a citizen to become a candidate.

answer The citizen–candidate model is discussed in section 5.3 of *Persson and Tabellini*.

In that model, each candidate is identified with her own most preferred policy : if a candidate whose most–preferred policy is x gets elected, then everyone knows that the policy x will get chosen.

It is assumed that people don't want to run. It's costly to become a candidate. If the cost of running is ϵ, then person i will get utility of $u_i(x) - \epsilon$ if she runs, and if a candidate with most–preferred policy x gets elected, where $u_i(x)$ is person i's utility from the policy x. If she does not run, and if a candidate with preferred policy y is elected, then person i's payoff will be $u_i(y)$.

So an outcome with exactly two candidates, i and j, will be an equilibrium if two conditions hold :

(a) no third candidate wants to enter

(b) neither (i) nor (j) wants to withdraw from the election (and save the cost ϵ of running)

Condition (b) implies that, if we have an equilibrium with 2 candidates, then each candidate must have a chance of winning : if candidate i were (for certain) the winner over candidate j, then candidate j would want to withdraw, since withdrawal would save him the cost ϵ, and not have any effect on the outcome.

If each candidate has a chance of winning, that means that half the voters must prefer one candidate's policy, and half the voters must prefer the other candidate's policy. Here a candidate's policy is her most–preferred policy, since all voters know that's what a candidate will choose once she's elected.

In a simple case, in which (1) voters' preferred policies are points on a line; (2) voters care only about the distance of a policy from their most–preferred policy ; (3) voters' most–preferred policy are arranged symmetrically over some interval $[-A, A]$ (so that the median voter has a preferred policy of 0), then the only possible situation in which each of the two candidates has a positive chance of winning is if the two candidates are lined up at equal and opposite distances from the median : candidate i has a preferred policy $-x$ and candidate j has a preferred policy x,

Assume as well that each candidate has an equal chance of winning, if the median voter is indifferent between them. If candidates care only about the distance between the actual policy, and their most–preferred policy, then the utility of a person with preferred policy z will be $B - |y - z|$ if policy y is actually chosen, where B is a constant.

So the payoff to the rightmost candidate will be

$$B - (0.5)(2x) - \epsilon = B - x - \epsilon \qquad (3-1)$$

if she runs, since with probability 0.5 the actually policy will be her own preferred policy x, and with probability 0.5 it will be the other candidate's preferred policy $-x$ (which is a distance $2x$ away from her own preferred policy). On the other hand, if she decides not to run, then the other candidate will win for sure, and her payoff will be

$$B - 2x \tag{3 - 2}$$

From expressions $(3 - 1)$ and $(3 - 2)$, the candidate will be willing to run if and only if expression $(3 - 1)$ exceeds expression $(3 - 2)$, or

$$x \geq \epsilon \tag{3 - 3}$$

But condition (a) must also hold if there is an equilibrium with only two candidates. The third candidate who has the best chance of winning is a candidate who has a preferred policy at the median, a preferred policy of 0 in this example. If she runs, she will get the votes of all people who are closer to the centre than to x or $-x$. If the voters' preferred policies are uniformly distributed over $[-A, A]$, that would mean that there would be x voters, out of a total electorate of $2A$ voters, who would vote for a candidate with a policy of 0 in a 3–candidate election : voters whose preferred policies lie in $(-x/2, x/2)$.

So a third candidate would win only if she got at least 1/3 f the votes, or $x/A > 1/3$, or

$$x > A/3 \tag{3 - 4}$$

A third candidate might still not want to run, even if she could win, if the cost of running were too high. The median voter would want to run, if she could win, only if

$$B - x < B - \epsilon \tag{3 - 5}$$

since the left side of expression $(3 - 5)$ is her payoff if she does not run (and a candidate at x or $-x$ wins – she doesn't care which), and the right side is her payoff if she runs and wins. Assuming that ϵ is small (small enough that $\epsilon < A/3$, condition $(3 - 5)$ will hold whenever the median voter has a chance of winning.

So in this special case, there will be a two candidate Nash equilibrium whenever the two candidates have positions $-x$ and x, with

$$\epsilon < x < A/3 \tag{3 - 6}$$

More generally, if all voters have single–peaked preferences, a two–candidate equilibrium will exist if and only if :

(1) the candidates' policies are on either side of the median, with the median voter indifferent between them

(2) the candidates' policies are far enough apart that the disutility of having the other candidate's policy (compared to her own) is at least twice as large as the cost of running

(3) the candidates' policies are close enough together that fewer than one–third of the voters prefer the median preferred policy to either of the candidates' policies

4. How much spending on public monuments should voters allow an elected official to undertake, in the following model of "retrospective voting"?

   Voters can observe exactly the amount r which the elected official spends on public monuments. Voters get no benefit at all from this spending, and care only about minimizing the amount of money which elected officials spend. Voters can punish the elected official (after the fact) by coordinating on a voting strategy, and voting against an official who spends more on public monuments than voters allow.

   The elected official places a value $V = 1$ on getting re–elected. The official also places a value of $6r - r^2$ on the amount she spends on public monuments (so that the official's payoff is $6r - r^2 + 1$ if she spends r dollars on public monuments and is re–elected, and $6r - r^2$ if she spends r dollars and is not re–elected). The largest possible amount of money which is available to be spent on public monuments is 5.

   answer This is a question about retrospective voting (*Persson and Tabellini*, section 4.4), but in a simplified setting, in which the politician's actions are observable, and in which there is no category of politician's spending which benefits voters.

   Since voters get no benefit from public spending r, they want r as small as possible. So they should try and induce the politician to curtail her spending, by voting for her only if she spends r^* or less, where r^* is the threshold level of spending which voters must choose.

   They want r^* to be small, but if they set it too small, then the politician may get such a low payoff from staying within the threshold ($r \leq r^*$) that she will choose not to get re–elected, spending as much as she pleases and forgoing the rent $V = 1$ that she would earn from being re–elected.

   If she does choose to ignore the voters, and to forget about re–elected, her payoff will be $6r - r^2$. So she should pick a level of spending r to maximize this expression. Since the derivative of this expression with respect to r is $6 - 2r$, she will maximize this payoff by choosing $r = 3$. Here the constraint that $r \leq 5$ does not bind, since the politician's own payoff from spending reaches a maximum at $r = 3$.

   If the politician chooses not to get re–elected, then her payoff will be $6(3) - (3)^2 = 9$.

   So if the voters coordinate on a strategy of voting for re–election only if the politician spends r^* or less, then they must set r^* so that

$$6r^* - (r^*)^2 + 1 \geq 9 \tag{4-1}$$

   If constraint $(4-1)$ does not hold, then the politician will simply pick $r = 3$ and not try to hold spending below r^*.

   So voters want to find the lowest level of r^* which satisfies $(4-1)$ as an equality. If $(4-1)$ holds with equality, it can be written

$$(r^*)^2 - 6r^* + 8 = 0 \tag{4-2}$$

   or

$$(r^* - 4)(r^* - 2) = 0 \tag{4-3}$$

From equation $(4-3)$, the smallest value for r which leaves the politician willing to seek re–election is $r^* = 2$. If voters commit to re–electing the politician if and only if she spends $r \leq 2$ on monuments, then the politician will get a payoff of

$$6(2) - 2^2 + 1 = 9 \qquad (4-4)$$

if she tries to satisfy the voters, which is as high as the payoff she would get if she were to ignore the voters and spend as much as she wants.

5. Suppose that an appointed public official gets to choose the budget for her department, but that her budget must be passed by a referendum among the voters. Suppose as well that there is some "reversion level" specified for public spending, if the budget is defeated. Would increasing this reversion level of spending decrease the amount of wasteful spending in the public sector? Explain briefly.

answer If the official wants to maximize the size of her budget, and if the reversion level of spending is relatively low, then the answer is "yes" : increasing the reversion level will decrease the size of the budget.

Suppose that voter i must pay, through his taxes, a share s_i of the cost of the department. Let the total cost of the department be $C(G)$, where G is the level of service provided by the department, and $C(\cdot)$ is an increasing, convex function : more services cost more, and the marginal cost of those services is a non–decreasing function of the level of service provided.

So if a level of services G is provided, and if the cost $C(G)$ of those services is paid for through taxes, then voter i's utility can be written as

$$U^i(w_i - s_iC(G), G) \qquad (5-1)$$

where the utility function $U^i(\cdot, \cdot)$ depends on the voter's after–tax income (w_i is her before–tax income) and on the level of public service provided by the department.

If preferences are convex, and if the total cost function $C(G)$ is convex, then expression $(5-1)$ increases with G up to some level G_i^* and then decreases with G.

[Proof : the derivative of utility with respect to G is

$$U_G^i - s_iC'(G)U_x^i \qquad (5-2)$$

where U_G^i and U_x^i are the marginal utility of public and private spending respectively. So utility will increase with G if and only if

$$\frac{U_G^i}{U_x^i} - s_iC'(G) > 0 \qquad (5-3)$$

Convex preferences imply that $\frac{U_G}{U_x}$ decreases as G increases, and the assumption that $C'' \geq 0$ implies that $s_iC'(G)$ does not decrease as G increases.]

Suppose that the reversion policy is some low level of spending G_0, in particular that $G_0 < G_i^*$. Then the payoff the voter would get if the referendum is defeated, and the reversion policy implemented, is

$$U^i(w_i - s_iC(G_0), G_0) \qquad (5-4)$$

If the official proposes a large budget, with $G > G_i^*$, then this voter will vote in favour of the budget in the referendum if (and only if)

$$U^i(w_i - s_iC(G), G) \geq U^i(w_i - s_iC(G_0), G_0) \tag{5 - 5}$$

So an official can get a large budget passed provided that inequality $(5-5)$ holds for at least half the voters in the referendum. If the reversion level G_0 increases, then the right side of $(5-5)$ will **increase**, for any voters i such that G_0 is "too low", that is for whom $G_0 < G_i^*$. So, for these voters, increases in the reversion level make it more appealing to vote "no" in the referendum. In particular, if the official has set her budget as high as possible, so that inequality $(5-5)$ holds as an equality for one "marginal" voter i, and as an inequality for m other voters, where there are $2m+1$ voters overall, and the budget just barely passed the referendum with the original G_0, then an increase in G_0 will cause the original budget to be defeated.

So the official must **decrease** the proposed level of spending in response to an increase in the reversion policy G_0. For the marginal voter i, if $(5-5)$ is still to hold as an equality, increases in G_0 must imply a decrease in G, whenever $G > G_i^* > G_0$.

6. Suppose that there are three voters in a legislature. Suppose as well that each of the three voters has single–peaked preferences over the possible alternatives.

   The three legislators are L, with the furthest–left preferred policy, M with the median of the preferred policies and R with the furthest–right preferred policy.

   The legislature's rules allow for a sequence of different proposals, possibly infinite. The rules of the legislature are : (1) L is chosen to make an initial proposal ; (2) if this initial proposal passes, it becomes the law, and nothing more happens ; (3) if the initial proposal (by L) is defeated, then M gets a chance to make a proposal ; (4) if M's proposal passes (after the initial proposal has been defeated), then M's proposal becomes the law, and nothing more happens ; (5) if M's proposal is defeated, then R gets to make a proposal ; (6) if R's proposal is passed (after the initial two proposals have been defeated), then R's proposal becomes law, and nothing more happens ; (7) if R's proposal is defeated, then we go back to step (1).

   A proposal requires a simple majority of the three votes in order to pass.Each legislator discounts the future at the rate $\delta < 1$ (so that her payoff from a policy passed in stage t is δ^{t-1} times her payoff from that same proposal if the proposal were passed in stage 1).

   What would happen in this legislature, (if all 3 legislators knew the rules just enumerated)?

   answer The short answer is : L will propose initially a policy which is just to the left of M's most–preferred policy. How far to the left of M most–preferred policy depends on the discount factor δ : the closer δ is to 1, the closer the proposal is to the median voter's most–preferred policy. Legislator M will support this initial proposal, and so the procedure ends after the initial proposal, with a policy being adopted which is very close to, but slightly left of, the median voter's most–preferred policy.

   The outcome described above must be the outcome in any subgame perfect Nash equilibrium to the game (played by the three legislators) defined by the procedure in the question.

Let λ, μ and ρ be the most–preferred policies of voters L, M and R respectively. They are points on a line, with $\lambda < \mu < \rho$.

Now consider what would be the outcome of the subgame which would arise if the first two proposals were defeated. That is, consider the game which begins in stage (5) of the procedure described, which would arise if L's and M's initial two proposals were defeated.

All three legislators are aware that this subgame will arise, if the first two proposals are defeated.

Let γ be the policy which gets chosen in equilibrium in this subgame : that is γ is the result which finally gets passed, if legislator R makes a proposal, and the L makes one if this proposal is defeated, and then M, and so on.

[The equilibrium might involve mixed strategies, and an uncertain outcome. Then let γ be the *expected* value of the policy chosen in the equilibrium to this subgame.]

This policy γ is located somewhere on this line (on which λ, μ and ρ are located). Suppose first that

$$\gamma \geq \mu \tag{6-1}$$

that is, that the outcome of this subgame is to the right of the median voter's preferred policy. Then legislator L will prefer strictly to have μ in stage (4) to having the game go to stage (5) : going to stage (5) results in a payoff of $\delta U^L(\gamma) < U^L(\gamma) \leq U^L(\mu)$, if $U^L(\cdot)$ is legislator L's utility function.

On the other hand, if

$$\gamma < \mu \tag{6-2}$$

then voter R would prefer strictly to have μ in stage (4) over having the game go on to stage (5).

[The above 2 paragraphs would still hold if the equilibrium to the subgame were uncertain, if legislators were risk averse. Their expected utility from the equilibrium to the subgame starting in stage (5) would be less than or equal to their utility from the expected value of the outcome, which is $U^i(\gamma)$.]

So it must be true that either L or R would prefer to have policy μ passed in stage (4), over letting the procedure continue to stage (5).

That means that M can get his most–preferred proposal μ passed in stage (4). That means he will choose to propose μ if the game gets to stage (3).

So voter M will vote for L's initial proposal, at the beginning of the game, only if it gives him a higher utility than letting the game go the next stage. Voter M's payoff, if the initial proposal is rejected, is therefore

$$\delta U^M(\mu) \tag{6-3}$$

and he will vote for the initial proposal if and only if it gives him a utility of at least $\delta U^M(\mu)$.

In the initial stage, voter L knows that her proposal can passed only if voter M supports it. She also knows that μ will be introduced by M, and passed, if her initial proposal is defeated. So what she has to do in the first stage is make the furthest–left proposal which M will support. That is, she will introduce a policy $\alpha < \mu$ such that

$$U^M(\alpha) = \delta U^M(\mu) \qquad (6-4)$$

[This assumes that δ is close enough to 1 that the solution to $(6-4)$ is to the right of λ. Otherwise, L would propose her favourite policy λ initially, since then $U^M(\lambda) \geq \delta U^M(\mu)$.]

The subgame perfect Nash equilibrium to the whole game then is :

(i) initially — and any other time she gets the chance to propose a policy — L proposes the policy $\alpha < \mu$ defined by equation $(6-4)$.

(ii) M votes for this proposal

(iii) if M somehow got to propose – which will not happen on the equilibrium path — he will propose μ

(iv) L would support M's proposal of μ if the game ever got to that stage (which it won't)

(v) if R somehow got to propose – which will not happen on the equilibrium path — she will propose $\gamma > \mu$ such that

$$U^M(\gamma) = \delta U^M(\alpha) = (\delta)^2 U^M(\mu) \qquad (6-5)$$

(vi) M would support R's proposal of γ if the game ever got to that stage (which it won't)

7. Discuss the appropriate level for the provision of education in the following model : 3 local jurisdictions, or 1 national jurisdiction?

All people have the same income, 100.

Each person regards 1 unit of the public good as being worth the same as a_i units of private consumption. (So a person's utility function is $c + a_i g$ if c is her private good consumption and g is her public good consumption.) The value a_i of the public good differs among people : there are 100 people for whom $a_1 = 0$, 100 people for whom $a_2 = 1$ and 100 people for whom $a_3 = 2$.

The total cost of providing g units of the public good to each of N people is $150g$. This is a total cost : the cost per person is $150g/N$.

People are perfectly mobile. The public good must be financed by a head tax. if the public good is provided at the national level, the quantity provided must be the same for everyone.

answer Three features about the economy described above :

(i) the technology of public good provision involves *economies of scale in population* ; since the cost per person falls with the number of people in the jurisdiction, providing the public good in one large jurisdiction will result in lower costs per person than providing it separately in 3 small jurisdictions

(ii) people differ in their taste for the public good

(iii) all three people regard the public good and the private good as *perfect substitutes* ; their indifference curves are straight lines (with slope a_i if the private good is graphed on the vertical axis and the public good on the horizontal)

Suppose first that the public good is provided at the local level. Because of the economies of scale (feature (i) above), it will be efficient to group together all people of the same type. If we have 3 jurisdictions, each containing 100 people, then the cost of the public good per person in each jurisdiction will be 3/2 per unit. That is, each jurisdiction, in choosing what level g_i to provide to the 100 people of type i, have a budget constraint

$$c_i = 100 - \frac{3}{2}g_i \qquad (7-1)$$

(since the population $N = 100$, and the total cost of the public sector is $150g$).

So jurisdictions 1 and 2, containing people of types 1 and 2, would choose not to provide any of the public good : the budget line in (g, c) space has a slope of 3/2 which is greater than the slope of people's indifference curves, so that each person of type 1 and 2 would prefer the point on the budget line defined by $(7-1)$ at which $c = 100$ and $g = 0$.

People of type 3 would prefer to be at the other end of the budget line, spending all their available income on the public good : they would choose $c_3 = 0$ and $g_c = 66.67$.

The utilities of the three groups, under local provision would be

$$U_1 = U_2 = 100 \qquad U_3 = 133.33 \qquad (7-2)$$

If the public good was provided at the national level, and all 300 people lived in the same jurisdiction, then the budget line relating private good consumption c_n and public good consumption c_n would be

$$c_n = 100 - \frac{1}{2}g_n \qquad (7-3)$$

Now groups 2 and 3 would both want to spend all available resources on public good provision. The question stated that all residents within any jurisdiction have to consume the same level g of the public good, and pay the same amount $150g/N$ for its provision.

So if groups 2 and 3 controlled public good provision at the national level, they would want to spend all the available income on public good provision : $g_n = 200$.

In this case, the utilities of the three groups, under national provision with $g_n = 200$ are

$$U_1^n = 0 \qquad U_2^n = 200 \qquad U_3^n = 400 \qquad (7-4)$$

So uniform national provision, at a level preferred by the majority, would make groups 2 and 3 better off, and group 1 worse off, than local provision.

Of course, lower levels of provision than the maximum $(g = 200)$ would be better for group 1, but worse for groups 2 and 3. But there is no level g of national provision which makes all 3 groups better off (than under local provision), since group 1 has to pay for the public sector, and derives no benefit from it.

Finally, if group 1 has veto power over any fiscal arrangements, they will never want to join with the other two groups. In that case, groups 2 and 3 might consider a middle level of jurisdiction. In a "province" containing 200 people, from groups 2 and 3, the cost per capita of the public good would be 3/4. In that 200-person province, all 200 residents would agree that the best policy would be to spend all the provincial income on public goods, so that $c = 0$ and $g = 133.33$. The type–1 people, in jurisdiction 1, would not join this province, and would choose to provide none of the public good. So utilities in this intermediate case would be

$$U_1 = 100 \qquad U_2^P = 133.33 \qquad U_3^P = 266.67 \tag{7-5}$$

So this intermediate arrangement —- 200 people in 1 jurisdiction and 100 in another — Pareto dominates the first case of local provision. But moving from this intermediate case to national provision benefits 200 people and harms the remaining 100.

8. What bill should a legislator propose, in the following situation?

The legislator has been chosen to propose the legislation for spending on parks in different districts. People benefit only from spending in parks in the district in which they live. The cost of all spending on parks will be divided equally among all the country's residents. Each district has a representative in the legislature, although different districts may have different populations. The value residents place on spending on a park in their district may also vary across districts.

A bill needs a simple majority to pass, and there will be no spending on parks at all if the bill is defeated.

answer Persson and Tabellini discuss legislative bargaining in this sort of framework of "distributive spending" in section 7.2.

Suppose that there are $2m+1$ districts represented in the legislature. To get a bill passed, the legislator proposing the bill needs the support of at least m other legislators. So the legislator must propose a bill which is preferred by at least m other legislators to the alternative, which here is no spending on parks in any district.

If g_i is the amount of spending on parks in district i, and P_i is the population of district i then the cost of the bill – to each district – is

$$t = \frac{sum_{i=1}^{2m+1} g_i P_i}{2m+1} \tag{8-1}$$

The preferences of each legislator might be represented by some function $U^i(w_i - t, g_i)$, where the first argument in the function is the district's residents' after–tax income (w_i is their before–tax income).

So increasing the amount of spending proposed in g_i in any district will make the bill less attractive to legislators in each of the other $2m$ districts, since their taxes t go up (according to $(8-1)$), and they get no benefits. This increase, however, will make the bill more attractive to the representative from district i, as long as spending is low enough so that

$$MRS_i \equiv \frac{U_g}{U_c} > \frac{P_i}{2m+1} \tag{8-2}$$

where U_g^i and U_c^i are the marginal utilities of parks and after–tax income respectively.

The representative from district i will vote for the bill if it gives her constituents more utility than they would get without the bill being passed. That is, she will support the bill if and only if

$$U^i(w_i - t, g_i) \geq U^i(w_i, 0) \qquad (8-3)$$

The legislator must propose some spending in at least m other districts, besides her own : equation $(8-3)$ can only hold if $g_i > 0$. In fact g_i must be high enough so that the benefits of the district's own parks exceed the district's share of the costs of the parks proposed everywhere.

The person proposing the bill, say the representative from district #1, wants to spend as little as possible on parks in other districts. So she should try and gain support from the minimum number of other districts, m of them. She should also spend no more than is necessary to gain the support in those m districts : she should try and have $(8-3)$ satisfied as an equality for the m districts for which she proposes $g_i > 0$.

If she has chosen a set of m districts in which to spend – call this set $\mathcal{M}$ — then the legislator from district 1 has the problem of choosing g_1, and g_i for each $i \in \mathcal{M}$ so as to maximize her district's own benefits

$$U^1(w_1 - t, g_1)$$

subject to the constraints that

$$U^i(w_i - t, g_i) = U^i(w_i, 0) \quad i \in \mathcal{M} \qquad (8-4)$$

$$g_i = 0 \quad i \notin \mathcal{M} \qquad (8-5)$$

and the definition $(8-1)$ of the taxes.

But she also has to figure out which m legislators from whom to get support. Since she gets support (only) by spending in legislators' districts, she should try and get support from districts for which (a) there is a strong taste for parks (so that U^i increases a lot for a relatively small g_i) and (b) the population is small (so that a given increase in g_i has a relatively small effect on the taxes t which every district must pay.