some sample exercises : Public Choice

1. The following table lists the way that 5 different voters rank five different alternatives.

   Is there a Condorcet winner under pairwise majority rule for this example?

   Explain briefly.

	voter #1	voter #2	voter #3	voter #4	voter #5
first choice	w	v	x	y	z
second choice	v	x	z	z	w
third choice	x	y	v	v	v
fourth choice	y	z	w	w	y
fifth choice	z	w	y	x	x

2. Prove that a Condorcet winner must exist under pairwise majority rule if all voters' preferences are single–peaked.

3. Give an example in which there is some arrangement of the alternatives such that (i) preferences are single–peaked for every voter except one ; and (ii) there is no Condorcet winner.

4. "Although cycles are a problem with pairwise majority rule in theory, they are not likely to arise in choices over public expenditure." Comment.

5. Suppose that policies could be represented by pairs of numbers (such as the sales tax rate, and the level of carbon emissions allowed). Suppose that each of 5 voters i had a preferred policy (x_i, y_i), and ranked policies by how far they were from her preferred policy (the further away from her preferred policy, the less she likes it).

   If the preferred policies of the 5 voters were $(1,1)$, $(1,5)$, $(3,3)$, $(5,1)$ and $(5,5)$, would there be a policy which defeats all others in a pairwise vote? If so, which one? If not, why not?

6. "If, for a given set of voters and alternatives, there exists a Condorcet winner, then the Condorcet winner will get the highest score using the Borda count." True or false? Explain briefly.

7. Give an example of a voting rule in which adding a "clone" (a new alternative which is very similar to one of the existing alternatives) could change the outcome.

8. If preferences over candidates for some elected office happened to be single–peaked, would the median of the most–preferred candidates win under plurality rule? Explain briefly.

9. Suppose that there are 11 voters of type 1, 12 voters of type 2, 13 voters of type 3, 14 voters of type 4 and 15 voters of type 5, with the following preference orderings over candidates :

	[11 people]	[12 people]	[13 people]	[14 people]	[15 people]
first choice	v	w	x	y	z
second choice	w	x	v	w	v
third choice	x	y	w	v	x
fourth choice	y	z	y	z	w
fifth choice	z	v	z	x	y

Describe the outcome of the following voting rules, for the population of voters described in the table :

(a) Plurality rule (the candidate with the most first–place votes wins).

(b) Plurality with a single run-off (if no candidate has a majority of the first–place votes, a second run–off election is conducted between the two candidates with the most first–place votes).

(c) "Hare system" : if no candidate has a majority of first–place votes, the candidate with the least first–place votes is eliminated, and the procedure repeated until there is a majority winner.

(d) "Coombs system" : if no candidate has a majority of first–place votes, the candidate with the most last–place votes is eliminated, and the procedure repeated until there is a majority winner.

(e) Borda count.

(f) "Nanson system" : do a Borda count, eliminate all candidates with Borda scores less than the average, and repeat until there is one candidate left

(g) "Baldwin system" : do a Borda count, eliminate the candidate with the lowest Borda score ; redo this process 3 more times until only one candidate is left

(h) "Black's system" : choose a Condorcet winner if there is one ; otherwise choose the candidate with the highest Borda score

10. Suppose that the social choice rule ranks y above x when the two voters' preferences are as listed in the table below. Suppose as well that the social choice rule is transitive, and obeys the Pareto principle ("unanimity") and the Independence of Irrelevant Alternatives.

Show that the social choice rule must always rank y above z whenever voter #2 ranks y above z.

	person #1	person #2
first choice	x	y
second choice	z	x
third choice	y	z

11. Which of the axioms of Arrow's Impossibility Theorem does the following social choice rule violate? The rule : if person #1 and person #2 have exactly the same rankings of the alternatives, then the social choice rule will exactly coincide with their ranking ; if their rankings are different in any way, then the social choice rule will coincide with person #3's ranking.

12. Sketch a proof of Arrow's Impossibility Theorem (without providing details on the steps in the proof)?

13. Show why the Borda count procedure is not strategy–proof.