

some sample exercises  
Public Choice : sections 4 – 8

1. What are the key assumptions needed to get the result that all parties will choose the median of the voters' most-preferred policies?

2. Is the following an equilibrium in the Hotelling–Downs model of vote maximizing parties, in which parties are free to pick any position they want in order to maximize the number of votes they get?

There are 4 parties. (The number of parties is fixed ; no new parties can enter.) The voters' most-preferred policies are distributed uniformly over the interval  $[-1, +1]$ , and each voter votes for the party with a platform closest to her own preferred policy.

One party chooses a platform just to the left of the point  $-0.5$  ; the second party chooses a platform just to the right of  $-0.5$  ; the third party chooses a platform just to the left of  $+0.5$  and the fourth party chooses a platform just to the right of  $+0.5$ .

3. Suppose that alternative policies in an election can be represented as points in two dimensions, and that voters always vote for the party with a platform closest to their own preferred policy.

One-third of the voters have a preferred policy at  $(-1, 0)$ , one-third of the voters have a preferred policy at  $(+1, +1)$ , and one-third of the voters have a preferred policy at  $(+1, +2)$ .

If there are exactly two parties, each of which is free to choose whatever platform it wishes, and each of which wants to miximize its chances of winning, could there be an equilibrium? If so, what is it? If not, why not?

4. Is the following an equilibrium in the “citizen–candidate” model, if voters preferred policies are points on a line, uniformly distributed between  $-1$  and  $1$ , if the cost of running in an election is  $0.1$ , the value of the prestige of being elected is  $0.2$ , and the cost of having a policy  $p$  in pace which is different from a person's own preferred policy  $x$  is  $(p - x)^2$ ?

Two candidates enter, one with a preferred policy of  $-0.5$ , and the other with a preferred policy of  $+0.5$ .

5. Calculate the equilibrium platforms chosen by vote-maximizing parties in the following probabilistic voting model.

Platforms are just points on the line. There are two parties. If party #1 chooses platform  $x$ , and party #2 chooses platform  $y$ , then the probability that **any** voter votes for party #1 is

$$\begin{aligned}
 P_1 &= 0 && \text{if } x^2 > y^2 + 1 \\
 P_1 &= \frac{y^2 - x^2 + 1}{2} && \text{if } -1 \leq y^2 - x^2 \leq 1 \\
 P_1 &= 1 && \text{if } y^2 > x^2 + 1
 \end{aligned}$$

6. What are the predictions of Niskanen's model of budget-maximizing bureaucrats?

7. What level of service provision  $X$  and what budget  $B$  should a government official propose in the following circumstances?

The official wants the highest level of service provision possible. But the budget  $B$  she requests must cover the cost  $C(X)$  of providing the service where

$$C(X) = X^2$$

She can propose any service level  $X$  and budget  $B$  which she wishes (provided that  $B \geq C(X)$ ). However, the proposal must pass a referendum of the voters.

If the proposal does not pass, the constitution dictates that a "basic" service level of  $X = 1$  will be implemented, with a budget of  $B = 1$ .

Each voter gets a total benefit of

$$W(X) = 12X - X^2$$

from a public service level  $X$ , and cares only about her total benefits  $W(X)$ , minus the budget  $B$  which is financed from his taxes.

8. "If voters can detect diversion of funds by elected officials, but only after the diversion has occurred, then they should commit to voting against any incumbent who has been detected diverting funds." True or false. Explain briefly.

9. Suppose that there are three city councillors on a committee, which gets to choose how to divide \$3000 in neighbourhood improvement funds among the three councillors' districts.

Any proposal on division of the money must be passed by a majority of the three-member committee. If it is defeated, then the funds remain unspent until the next session of the committee. Each session, one councillor is chosen at random to make a proposal.

What happens in the first session, if councillor #1 has just been chosen to make the first proposal?

10. Suppose that the software industry in Canada is concentrated in a few different areas. Suppose as well that any legislature concerning software regulation must be proposed by a special committee, with 5 members, and that leaders of the major parties get to choose which of their members of parliament serve on this committee.

Explain why this institutional arrangement may lead to a level of protection of software copyrights which is stronger than the level of copyright protection which maximizes the net benefits of all Canadians.

11. What are the key assumptions which ensure that public output provision by the local public sector will be efficient, even if people differ in their taste for this local public output, and if no-one knows anyone else's tastes.

12. Should fire protection be provided at the national or regional level in the following example?

The total cost of  $X$  units of fire protection is  $30X$ , **regardless of the number of people being protected**. This cost is divided equally among all people in the jurisdiction. Fire protection must be provided at a uniform level (one level for the whole jurisdiction).

There are two regions in the nation. Region 1 has 3 people, each of whom derives total benefits of

$$W_1(X) = 20X - X^2$$

from a level  $X$  of fire protection. Region 2 has 2 people, each of whom derives total benefits of

$$W_2(X) = 17X - X^2$$

from a level  $X$  of fire protection.

People care about their net benefit  $W_i(X)$ , minus their share  $30X/N$  of the costs, where  $N$  is the number of people sharing the cost.

The jurisdiction's level of fire protection is decided by pairwise majority rule (whether the jurisdiction is a region, or the nation), so that the preferences of the median voter prevail.

13. Suppose that spending  $X$  dollars in district  $i$  yields benefits of  $\ln X$  (i.e. the natural logarithm of  $X$ ) to residents of district  $i$ , and no benefits at all to residents of any other district.

Costs of all spending must be divided equally among all districts.

Compare the total amount of spending (in all districts) in the following 2 situations : (i) a "bare-minimum" coalition forms, of representatives of a bare majority of the districts, and this coalition proposes spending legislation ; (ii) a "universalist" norm prevails, in which spending proposal gets approved by representatives of all the districts.

14. Re-do the previous question, when the benefit to a district of  $X$  dollars spending is not  $\ln X$  but : (a)  $2\sqrt{X}$  or (b)  $100 - \frac{1}{X}$ .

15. If public spending on coal-fired power plants in a province produced a **negative** spillover — more public spending on the plants in province  $i$  increases the damage done to residents of other provinces — what sort of transfer programme from the federal governments to the provinces would help correct this spillover?

16. Suppose that the income elasticity of demand for education is 1, and that education spending in Ontario equals 5% of total provincial income.

Theoretically, what would be the effect on Ontario's public education of a lump-sum grant of \$10,000,000 from the federal government to the Ontario government, which must be spent on public education?

How does the "flypaper effect" modify this theoretical prediction?

17. How much rent dissipation is there in a contest in which 3 identical lobbyists choose how much to spend trying to win a prize worth  $I$ , in which the probability that lobbyist  $i$  wins the prize is

$$\pi_i = \frac{I_i}{I_1 + I_2 + I_3} \quad ?$$

18. Give an example of a rent-seeking contest in which there is no equilibrium (at least in pure strategies) when there are two lobbyists competing for the prize.

19. What would be the equilibrium in the following lobbying model?

The government promises to award a vacant piece of property worth  $R$  dollars to some firm.

Firms compete for the prize by pledging to use part of the property for a local entertainment centre. Each firm  $i$  makes a proposal to spend  $E_i$  on the local entertainment centre, should they win the property.

The government announces it will award the property to the firm which pledges the most money  $E_i$  for the local entertainment centre. The winning firm has to honour its pledge, and spend the  $E_i$  it promised. The losing firms get nothing, and don't have to spend any money on the entertainment centre. (That is, the money is pledged conditional on the firm getting the property.)

20. Why do industries in decline (such as some American “rust-belt” manufacturing) still appear to have some much political power?