answers to some of the sample exercises : sections 4 – 8   Public Choice

Ques 1 What are the key assumptions needed to get the result that all parties will choose the median of the voters' most–preferred policies?

answer The result described is often referred to as the Hotelling principle of minimum differentiation, applied by Downs to political models. The key assumptions are : (i) there are exactly 2 parties ; (ii) each party can choose whatever policy it wishes ; (iii) policies can be represented by points on a line ; (iv) voters have single–peaked preferences over these points on a line ; (v) voters always vote (and always vote for the policy they like best) ; (vi) parties care only abut maximizing the number of votes they get.

The last 2 assumptions are not as crucial ; the minimum differentiation result might hold even if voters could get "discouraged", and even if parties had some ideology.

Ques 2 Is the following an equilibrium in the Hotelling–Downs model of vote maximizing parties, in which parties are free to pick any position they want in order to maximize the number of votes they get?

There are 4 parties. (The number of parties is fixed ; no new parties can enter.) The voters' most–preferred policies are distributed uniformly over the interval $[-1, +1]$, and each voter votes for the party with a platform closest to her own preferred policy.

One party chooses a platform just to the left of the point -0.5 ; the second party chooses a platform just to the right of -0.5 ; the third party chooses a platform just to the left of $+0.5$ and the fourth party chooses a platform just to the right of $+0.5$.

answer What must be checked is that no party can gain votes by changing its platform. As is, each party gets 25 percent of the votes.

If the leftmost party moved left, to some platform $-x$, with $x > 0.5$, then they would lose votes ; half the people between $-x$ and -0.5 would switch their vote to the second party (the one just to the right of -0.5). And if the leftmost party moved anywhere else, they could not gain votes : if they moved to some platform $-x$ between -0.5 and 0, then they would get the vote of half the voters between $-x$ and $+0.5$, and half the voters between -0.5 and $-x$. That's exactly 25 percent of the voters, so they don't gain by the switch. Similarly, a move to any platform x between 0 and 0.5 will get them exactly 25 percent of the votes, and a move anywhere to the right of $+0.5$ will get them less than 25 percent of the votes.

So any change of platform by the first party either leaves it with the same number of votes as before 25 percent of them), or loses them votes. The first party has no incentive to change its platform.

Similarly any move by the second party either yields it the same number of votes as it already has (if it moves to a platform in $(-0.5, +0.5)$, or loses it votes (if it moves to a platform in $(-1, -0.5)$ or in $(0.5, 1)$). And the third and fourth party have exactly analogous calculations.

Since none of the four parties can gain votes by a unilateral change in policy, the platforms described do constitute an equilibrium in the Hotelling–Downs model with 4 parties.

Ques 3 Suppose that alternative policies in an election can be represented as points in two dimensions, and that voters always vote for the party with a platform closest to their own preferred policy.

One–third of the voters have a preferred policy at $(-1, 0)$, one–third of the voters have a preferred policy at $(+1, +1)$, and one–third of the voters have a preferred policy at $(+1, +2)$.

If there are exactly two parties, each of which is free to choose whatever platform it wishes, and each of which wants to maximize its chances of winning, could there be an equilibrium? If so, what is it? If not, why not?

answer There is no equilibrium here. The way the preferred policies are arranged, no policy is a "median in all directions". And if there is no median in all directions, the results from section 1 of the course say that there will be cycles : for **any** policy (x, y), there is some other policy (w, z) which is preferred to (x, y) by two–thirds of the voters.

So whatever are the platforms of the two parties, one party can gain by switching its platform to one which can defeat the other party, and then the other party can switch to a platform which defeats the first party, and so on.

Ques 4 Is the following an equilibrium in the "citizen–candidate" model, if voters preferred policies are points on a line, uniformly distributed betwen -1 and 1, if the cost of running in an election is 0.1, the value of the prestige of being elected is 0.2, and the cost of having a policy p in pace which is different from a person's own preferred policy x is $(p - x)^2$?

Two candidates enter, one with a preferred policy of -0.5, and the other with a preferred policy of $+0.5$.

answer If this is to be an equilibrium, two properies must be checked : (i) both candidates who are running prefer to enter (rather than stay out), and (ii) no third candidate wants to enter.

Turning to the second property first, note that no new candidate can win this election, given the two candidate who already enter. A candidate at the dead centre of the policy line (with a preferred policy $x = 0$) would get votes of all voters in $[-0.25, +0.25]$; that's 25 percent of the voters, fewer than either of the already–entered candidates would get. (They'd each get 37.5 percent : the voters in $[-1, -0.25)$ and those in $(+0.25, +1]$ respectively.)

So a candidate from the dead centre cannot enter and win. Neither can any candidate just to the left of centre, or just to the right. And any candidate more extreme than -0.5 or $+0.5$ would get less than 25 percent of the vote, and lose.

No new entrant can win. And the entry of a candidate from the left of the spectrum (in the interval $[-1, 0)$) would just ensure the guaranteed election of the rightist candidate with position $+0.5$. So any entrant would just guarantee the election of the existing candidate whom she least prefers.

Property (ii) of the equilibrium therefore must hold : with existing candidates at -0.5 and $+0.5$, no new candidate wants to enter.

How about property (i)? Given the entry of the two candidates, each will win with a fifty percent probability. So the expected payoff of the rightist candidate is

$$0.5(0.2) - (0.5)([0.5 - (-0.5)]^2) - 0.1 = -0.5$$

The first term on the left is the value of the candidate's expected prestige ; the second term is the expected cost of having the other candidate win, resulting in a policy the rightist candidate does not like ; the third term is the cost of entry.

But if the rightist candidate did not enter, then the leftist candidate would win for sure, so that the rightist's expected payoff would be

$$-([0.5 - (-0.5)]^2) = -1$$

That's a higher expected cost (or a lower expected payoff) than she gets if she runs. So, given the presence of the leftist candidate, she would rather be running than out of the race.

A similar calculation holds for the leftist candidate.

Property (i) of equilibrium therefore holds : each candidate in the race wants to stay in the race.

Therefore, it is an equilibrium for exactly 2 candidates to enter, one at -0.5 and the other at $+0.5$.

Ques 5 Calculate the equilibrium platforms chosen by vote–maximizing parties in the following probabilistic voting model.

Platforms are just points on the line. There are two parties. If party #1 chooses platform x, and party #2 chooses platform y, then the probability that **any** voter votes for party #1 is

$$P_1 = 0 \qquad \text{if} \quad x^2 > y^2 + 1$$

$$P_1 = \frac{y^2 - x^2 + 1}{2} \qquad \text{if} \quad -1 \leq y^2 - x^2 \leq 1$$

$$P_1 = 1 \qquad \text{if} \quad y^2 > x^2 + 1$$

answer If party 1 chooses its platform so as to maximize its expected votes, then if $-1 < y^2 - x^2 < +1$, it is choosing x so as to maximize $\frac{1}{2}(y^2 - x^2 + 1)$. Maximizing this probability with respect to the platform x has a first–order condition

$$-x = 0$$

So party #1's best platform is $x = 0$ (if $-1 \leq y \leq +1$).

But party #2 wishes to maximize its probability of winning, which is $1 - P_1$, so it picks y so as to maximize $1 - \frac{1}{2}(y^2 - x^2 + 1)$. Here the maximization has first–order condition

$$-y = 0$$

so that party #2's best platform is $y = 0$.

Therefore, the equilibrium here is the "Hotelling–like" result that both parties pick the identical platform, $x = y = 0$, since voters want the platforms to be close to their ideal point 0.

Ques 7 What level of service provision X and what budget B should a government official propose in the following circumstances?

The official wants the highest level of service provision possible. But the budget B she requests must cover the cost $C(X)$ of providing the service where

$$C(X) = X^2$$

She can propose any service level X and budget B which she wishes (provided that $B \geq C(X)$). However, the proposal must pass a referendum of the voters.

If the proposal does not pass, the constitution dictates that a "basic" service level of $X = 1$ will be implemented, with a budget of $B = 1$.

Each voter gets a total benefit of

$$W(X) = 12X - X^2$$

from a public service level X, and cares only about her total benefits $W(X)$, minus the budget B which is financed from his taxes.

answer The "basic" service level gives each voter a total benefit of $W(1) = 12(1) - 1^2 = 11$, and costs $1^2 = 1$, so that the net welfare $W(X) - B$ to the voter of this basic proposal is 10.

The official therefore must offer the voter a net welfare of at least 10, if she wants her proposal passed.

The official should set her proposed budget B equal to the actual cost $C(X^*)$ of her proposed service level X^*. (The budget must be at least $C(X^*)$, and if it's any higher, then the high cost is jeopardizing the chance of the proposal being passed.)

So the official should find a service level X^* which is as high as possible, subject to the voter preferring it to the basic alternative. That constraint, that $W(X^*) - C(X^*) \geq W(1) - 1$ can be written

$$12(X^*) - (X^*)^2 - (X^*)^2 \geq 10$$

or

$$2(X^*2) - 12X^* + 10 \leq 0 \qquad (q7)$$

The quadratic equation on the left side of $(q7)$ can be written

$$2(X^* - 1)(X^* - 5)$$

So constraint $(q7)$ is satisfied as long as the proposed service level X^* is between 1 and 5. Therefore the largest service level X^* which she can get passed is $X^* = 5$, and the associated budget which covers the cost of this service level is $B^* = 25$.

Ques 9 Suppose that there are three city councillors on a committee, which gets to choose how to divide $3000 in neighbourhood improvement funds among the three councillors' districts.

Any proposal on division of the money must be passed by a majority of the three–member committee. If it is defeated, then the funds remain unspent until the next session of the committee. Each session, one councillor is chosen at random to make a proposal.

What happens in the first session, if councillor #1 has just been chosen to make the first proposal?

answer Councillor #1 wants as much as possible of the money to go to her own district. But she needs to get at least one of the other two councillors to support her proposal. She has no need to get the support of both of the other councillors : one of them is enough to get a proposal passed.

So she should pick one of the other two councillors (say councillor #2), and try to allocate just enough of the funds to that councillor's district so as to get him to vote for the proposal.

Now councillor #2 knows that if he doesn't support the proposal, the funds will stay there until the next session. Since each councillor has an equal chance of being in charge of the next session (and of any subsequent sessions, if no proposal gets passed in the second session), councillor figures that "on average", he'd get one–third of the funds if he waited until the next session.

So to get councillor #2's support, councillor #1 must allocate at least one–third of the funds to councillor #2's district. Otherwise councillor #2 would rather vote against the proposal and wait until the next session.

Therefore, councillor #2 proposes allocating the smallest amount of funds possible to councillor #2's district, subject to getting councillor #2's vote. The rest of the money she can allocate to her own district.

So what councillor #1 proposes is spending $2000 of the money in her own district, and the remaining $1000 in councillor #2's district.

Ques 12 Should fire protection be provided at the national or regional level in the following example?

The total cost of X units of fire protection is $30X$, **regardless of the number of people being protected**. This cost is divided equally among all people in the jurisdiction. Fire protection must be provided at a uniform level (one level for the whole jurisdiction).

There are two regions in the nation. Region 1 has 3 people, each of whom derives total benefits of

$$W_1(X) = 20X = X^2$$

from a level X of fire protection. Region 2 has 2 people, each of whom derives total benefits of

$$W_2(X) = 17X - X^2$$

from a level X of fire protection.

People care about their net benefit $W_i(X)$, minus their share $30X/N$ of the costs, where N is the number of people sharing the cost.

The jurisdiction's level of fire protection is decided by pairwise majority rule (whether the jurisdiction is a region, or the nation), so that the preferences of the median voter prevail.

answer The median voter in a jurisdiction wants to pick a level of fire protection X so as to maximize her net benefits (net of her share of the costs)

$$AX - X^2 - \frac{30X}{N}$$

where $A = 20$ for residents of region 1 and $A = 17$ for residents of region 2, and N is the number of people in the jurisdiction. Maximizing these net benefits leads to a first–order condition

$$A - 2X - \frac{30}{N} = 0$$

or

$$X = \frac{A - \frac{30}{N}}{2} \qquad (q12)$$

If the fire protection is provided at the regional level, then $A = 20$ and $N = 3$ in region 1, and $A = 17$ and $N = 2$ in region 2. So under regional provision we have

$$X_1 = 5$$

$$X_2 = 1$$

and the residents of the 2 regions have net benefits of

$$NB_1^S = 20(5) - 5^2 - \frac{(30)(5)}{3} = 25$$

$$NB_2^S = 17(1) - 1^2 - \frac{(30)(1)}{2} = 1$$

where the superscript S denotes "separate" regions.

If fire protection is provided (uniformly) at the national level, residents of region 1 get to choose the level of provision, since they are in the majority. Those residents choose a level X which satisfies their maximization condition ($q12$), with $A = 20$ and $N = 5$ (since costs are shared among all 5 residents of the nation), so that

$$X_N = 7$$

which means that, under uniform national provision, the regions' residents' net benefits are

$$NB_1^N = 20(7) - 7^2 - \frac{(30)(7)}{5} = 49$$

$$NB_2^S = 17(7) - 7^2 - \frac{(30)(7)}{5} = 28$$

where the superscript N denotes "national" provision.

Here all residents are better off under national provision : the cost savings from economies of scale in population outweigh the taste differences between regions.

Ques 13 Suppose that spending X dollars in district i yields benefits of $\ln X$ to residents of district i, and no benefits at all to residents of any other district.

Costs of all spending must divided equally among all districts.

Compare the total amount of spending (in all districts) in the following 2 situations : (i) a "bare–minimum" coalition forms, of representatives of a bare majority of the districts, and this coalition proposes spending legislation ; (ii) a "universalist" norm prevails, in which spending proposal gets approved by representatives of all the districts.

answer

Let N be the total number of districts represented in the legislature, and let M be the number of districts represented in the coalition. With a bare majority in the coalition, M is the largest integer greater than $N/2$; with a universalist norm, $M = N$.

So whether majoritarian or universalist, a representative's district gets a net benefit of

$$\ln X - \frac{M}{N} X \tag{q13}$$

if each district in the coalition gets X dollars of spending allocated to it, each district outside the coalition gets no spending, and all spending is divided equally among all N districts.

maximizing expression $(q13)$ with respect to X means setting its derivative with respect to X equal to 0 ; the level of spending which maximizes coalition members' net benefits is the solution to

$$\frac{1}{X} - \frac{M}{N} = 0$$

or

$$X = \frac{N}{M}$$

The bigger is the coalition, the smalling is the level of spending in each district in the coalition (since there are fewer districts outside the coalition to get stuck with part of the cost).

Total government spending is MX : X dollars in each of the M districts represented in the coalition. Since $X = \frac{N}{M}$, therefore

$$MX = N$$

regardless of the size of a coalition.

In this example, majoritarian and universalist norms lead to the same amount of total spending (although a different distribution of spending across districts). The larger coalition spends in more districts, but it spends less in each district in which there is spending. In this example, these two effects exactly offset.

Ques 14 Re–do the previous question, when the benefit to a district of X dollars spending is not $\ln X$ but : (a) $2\sqrt{X}$ or (b) $100 - \frac{1}{X}$.

answer In case (a), now the coalition maximizes

$$2\sqrt{X} - \frac{M}{N} X \tag{q14a}$$

so that the first–order condition for maximization is

$$\frac{1}{\sqrt{X}} - \frac{M}{N} = 0$$

or

$$X = [\frac{N}{M}]^2$$

meaning that total spending in all M districts is

$$MX = \frac{N^2}{M}$$

In case (b), the coalition maximizes

$$100 - \frac{1}{X} - \frac{M}{N}X \tag{14b}$$

with first–order condition

$$\frac{1}{X^2} - \frac{M}{N} = 0$$

or

$$X = \sqrt{\frac{N}{M}}$$

meaning that total spending in all M districts in the coalition is

$$MX = \sqrt{MN}$$

In case (a) total spending is higher under the smaller bare–majority coalition than under universalism, but in case (b) the reverse is true.

Ques 15 If public spending on coal–fired power plants in a province produced a **negative** spillover — more public spending on the plants in province i increases the damage done to residents of other provinces — what sort of transfer programme from the federal governments to the provinces would help correct this spillover?

answer A negative spillover is like a negative externality : if it ignores the effects of the spillover, a provincial government will choose an inefficiently high level of the spillover–causing activity.

So if the federal government wanted to use a transfer programme to reduce public spending on coal–fired plants towards the efficient level, it should be taxing provincial expenditure on coal–fired plants, rather than subsidizing it.

Two ways of doing this would be (i) a general transfer to each province, where the amount of the transfer decreased with the amount of provincial expenditure on coal–fired power plants ; (ii) a specific transfer for electricity generation, where the size of the transfer increased with each provincial **reduction** in the use of coal–fired plants.

Ques 16 Suppose that the income elasticity of demand for education is 1, and that education spending in Ontario equals 5% of total provincial income.

Theoretically, what would be the effect on Ontario's public education of a lump–sum grant of $10,000,000 from the federal government to the Ontario government, which must be spent on public education?

How does the "flypaper effect" modify this theoretical prediction?

answer In theory, a lump–sum grant which must be spent on education should have the same effect on Ontario's spending as a lump–sum grant with no strings attached. That's because Ontario can use the entire $10,000,000 on public education, but then decrease its own spending (from its own tax revenues) on education, and divert that money to other expenditures (or back to residents in the form of lower taxes).

So, in theory, the question is : how much would education spending in Ontario increase if Ontario's total income increase by $10,000,000? Let E be spending on education in Ontario, and Y the total income of Ontario. The income elasticity of education spending equalling 1 is the same thing as

$$\frac{\partial E}{\partial Y}\frac{Y}{E} = 1$$

or

$$\frac{\partial E}{\partial Y} = \frac{E}{Y}$$

Since the question stated that education spending was 5% of total income in Ontario, therefore $\frac{E}{Y} = 0.05$ and so

$$\frac{\partial E}{\partial Y} = 0.05$$

and so a $10,000,000 lump–sum grant to education should increase education spending by 5% of the grant, or $500,000.

Of course, residents of Ontario pay taxes to the federal government. So the cost of this grant is probably coming, at least in part, from increased federal taxes paid by Ontarians. If politicians and voters take the cost of these taxes into account, they'll realize that they really have not seen there income increase by $10,000,000 because of the grant, but by $10,000,000 minus the taxes paid in Ontario to pay for the grant. If Ontarians pay 40 percent of federal taxes, then rational voters should really getting only $6,000,000 since $4,000,000 of the grant comes from taxes collected from themselves. In this case, education spending would increase by 5 percent of the net gain, or only $300,000

The flypaper effect says that money sticks where it lands. The theory predicts that the Ontario government will respond to the $10,000,000 grant by reducing its own spending on education by $10,000,000 - 500,000 = 9,500,000$ dollars ($9,700,000 if they realize that they're paying for 40 percent of the grant). The flypaper effect says that Ontario will reduce its own education spending by much less than that, and so will wind up increasing its overall education expenditure by much more than $500,000.

Ques 17 How much rent dissipation is there in a contest in which 3 identical lobbyists choose how much to spend trying to win a prize worth I, in which the probability that lobbyist i wins the prize is

$$\pi_i = \frac{I_i}{I_1 + I_2 + I_3} \qquad ?$$

answer Lobbyist i wants to maximize her expected payoff, which is the expected value of the prize she wins, minus her lobbying expenses, $\pi_i R - I_i$, if R is the value of the prize. If $\pi_i = \frac{I_i}{I_1+I_2+I_3}$, then maximization of this expected payoff has a first–order condition

$$\frac{1}{I_1 + I_2 + I_3}R - \frac{I_i}{(I_1 + I_2 + I_3)^2}R - 1 = 0 \qquad (q17)$$

If all 3 lobbyists are identical, then in equilibrium, each lobbyist will choose the same level of lobbying expenditure $I_i = I$, and each will win the prize with probability $\pi_i = 1/3$ so that equation $(q17)$ becomes

$$\frac{1}{3I}R - \frac{I}{9I}R - 1 = 0$$

or

$$\frac{2}{9I}R = 1 = 0$$

meaning that

$$I = \frac{2}{9}R$$

(which is just equation (15.4) of Mueller, when $r = 1$ and $n = 3$).

In equilibrium, each of the three lobbyists spends 2/9 of the prize's value, in lobbying efforts to obtain the prize. Overall then, the three of them spend 3(2/9) of the value of the prize : 2/3 of the value is dissipated in the lobbying efforts.

Ques 18 Give an example of a rent–seeking contest in which there is no equilibrium (at least in pure strategies) when there are two lobbyists competing for the prize.

answer

This phenomenon can happen when there are strongly **increasing returns to scale** in lobbying expenditure : slightly more spending than the other lobbyist lads to a much higher chance of winning.

If the chance of lobbyists i winning is

$$\pi_i = \frac{I_i^r}{I_1^r + I_2^r}$$

where r is some positive constant, then lobbyist 1 chooses her lobbying expenditure I_1 so as to maximize

$$\pi_1 R - I_1 = \frac{I_1^r}{I_1^r + I_2^r}R - I \qquad (q18)$$

if R is the value of the prize for which they are lobbying.

Maximization of $(q18)$ with respect to lobbying expenditure I_1 has a first–order condition

$$rI_1^{r-1}\frac{1}{I_1^r + I_2^r} - rI_1^{r-1}\frac{I_1^r}{(I_1^r + I_2^r)^2} - 1 = 0$$

or

$$rR\frac{I_1^r}{I_1^r + I_2^r}[\frac{1}{I_1}](1 - \frac{I_1^r}{I_1^r + I_2^r}) - 1 = 0 \qquad (q18a)$$

Lobbyist 2 solves an analogous problem. If there were a symmetric equilibrium, in which $I_1 = I_2 = I$, then equation $(q18a)$ becomes

$$I_1 = I_2 = \frac{r}{4}R \qquad (q18b)$$

which is just equation (15.4) in Mueller, with $n = 2$.

So each lobbyist would win the prize with probability 0.5 in a symmetric equilibrium in which $I_1 = I_2 = I$. But if $r > 2$, then the amount of lobbying expenditure I in equation $(18b)$ will exceed $R/2$: the expenditure on lobbying is greater than the expected payoff.

This can't be an equilibrium : lobbyists always have the option of not lobbying (setting $I_i = 0$). So in equilibrium their lobbying expenditures cannot exceed their expected winnings ; otherwise they are better off not lobbying.

So if $r > 2$, there can be no symmetric equilibrium in pure strategies when two lobbyists compete for a prize.

Ques 19 What would be the equilibrium in the following lobbying model?

The government promises to award a vacant piece of property worth R dollars to some firm.

Firms compete for the prize by pledging to use part of the property for a local entertainment centre. Each firm i makes a proposal to spend E_i on the local entertainment centre, should they win the property.

The government announces it will award the property to the firm which pledges the most money E_i for the local entertainment centre. The winning firm has to honour its pledge, and spend the E_i it promised. The losing firms get nothing, and don't have to spend any money on the entertainment centre. (That is, the money is pledged conditional on the firm getting the property.)

answer The rules here define an **auction** : the pledges E_i are like bids, and the government has promised to award the prize to the high bidder, and to collect the bid (only) from the high bidder.

Unlike the "Tullock contests" of section 15.1 in Mueller, only winners pay here.

So the payoff to firm i is $R - E_i$ if its pledge E_i is more than any other firm's pledge, and 0 if its pledge is not the highest.

Could we have an equilibrium in which the highest pledge were less than R? Suppose that firm #1 has pledged $E_1 < R$, and that this pledge is the highest, or tied for the highest. There are two cases to consider :

case (i) $E_2 < E_1$. That means that firm #2 won't get the property, and has a payoff of 0. This can't be an equilibrium, since firm #2 could win the property for sure by pledging E_1 plus 1 cent : and as long as $E_1 < R$, firm #2 is better off winning the property (and getting a net payoff of $R - E_1 - 0.01$) than letting firm #1 win.

case (ii) $E_2 = E_1$. Now firm 2 has a chance of winning of 0.5 (if it's only tied with firm 1), or of $1/n$ (if it's tied with firm #1 and with $n - 2$ other firms). In either case, if $E_1 < R$, then firm #2 can do better by upping its pledge slightly, guaranteeing that it will the prize for sure (and an expected payoff of $(R - E_1 - 0.01)$) which is better than winning it with probability 0.5, with an expected payoff of $(0.5)(R - E_1)$ (or less, if there are more than 2 firms tied).

So here the pledges will completely dissipate the rents,. The "winner–takes–all" characteristic of the allocation rule means that the highest "bid" will be very close to the total value of the prize.