## distributive spending

(Persson and Tabellini, pp. 161-168)
what's new :

1. a (single) national legislature chooses public output in each "district"
2. the level of public output does not have to be uniform : it can vary across districts

BUT : taxes must be the same everywhere

## notation

$g^{J}$ : spending (per capita) in district $J$
$N^{J}$ : population in district $J$ - fixed (no mobility) $N$ : total national population $\left(=\sum_{J=1}^{Z} N^{J}\right)$
$y$ : income (same for everyone)
$\tau$ : tax rate

$$
\begin{equation*}
W^{J}=y-\tau+H\left(g^{J}\right) \tag{1}
\end{equation*}
$$

with $H^{\prime}>0$ and $H^{\prime \prime}<0$

## budget constraint

$$
\begin{equation*}
\tau N=\sum_{J=1}^{Z} g^{J} N^{J} \tag{2}
\end{equation*}
$$

so cost of public expenditure in district $J$ is proportional to the population (constant returns to scale in population)
tax rate

$$
\begin{equation*}
\tau=\frac{\sum_{J=1}^{Z} g^{J} N^{J}}{N} \tag{3}
\end{equation*}
$$

so raising $g^{J}$ by $\$ 1$ raises everyone's tax bill by $\frac{N^{J}}{N}$ dollars

## common pool problem

if, somehow, people in district $J$ got to pick spending $g^{J}$ in their own district (and people in every other district got to decide the level of spending in their own district) with taxes still paid nationally
then voters in district $J$ would most prefer a level of spending $g^{J, D}$ such that

$$
\begin{equation*}
H^{\prime}\left(g^{J, D}\right)=\frac{N^{J}}{N}<1 \tag{4}
\end{equation*}
$$

## too much spending?

$g^{J, D}$ is bigger than the level of public spending $g^{*}$ which residents in a district would choose if each district paid for its own spending

$$
\begin{equation*}
H^{\prime}\left(g^{*}\right)=1 \tag{5}
\end{equation*}
$$

with national taxes, most of the cost of $g^{J}$ is being passed on to residents of other districts, so district $J$ people would choose a high level of $g^{J}$ if they could, since someone else is paying for most of it

## a national legislature

each district has one member in a national legislature (assume the number $Z$ of districts is odd)
and spending per capita $g^{J}$ is not chosen by district $J$ 's own member : it must be decided by the whole legislature
increases in $g^{J}$ benefit only district $J$; and harm residents of every other district (since everyone shares in the cost)
so the only way any spending will get passed is through formation of some sort of coalition ("log-rolling")

## a minimal coalition

if the status quo were no spending at all $(\overline{\mathbf{g}}=(0,0, \ldots, 0))$ then a proposal needs to have $g^{J}>0$ for at least half the districts if it is to have any chance of passing
but if a proposal has spending in exactly $\frac{Z+1}{2}$ districts, (and enough spending to get support from each of those districts), then there is no point in trying to attract any more districts into the coalition

## bargaining model

status quo: some policy vector $\left(\bar{g}^{1}, \bar{g}^{2}, \ldots, \bar{g}^{Z}\right)$
representative from district a gets chosen to write the proposal ("agenda setter")
district a representative makes some proposal $\left(g^{1}, g^{2}, \ldots, g^{Z}\right)$
all districts vote : if more than $50 \%$ vote for the proposal $\mathbf{g}$, then the proposal is passed (and everyone pays taxes of
$\tau=\frac{\sum_{J=1}^{Z} g^{J} N^{J}}{N}$ ) and if more than $50 \%$ vote against the proposal, then the status quo $\left(\bar{g}^{1}, \bar{g}^{2}, \ldots, \bar{g}^{Z}\right)$ is implemented (with taxes
of $\bar{\tau}=\frac{\sum_{J=1}^{z} \bar{g}^{J} N^{J}}{N}$ )

## agenda setter's problem

representative of district a wants a large $g^{a}$ for her district (since most of it is paid for by people in other districts)
but she has to get her bill passed, which means "buying" the votes of at least $\frac{Z-1}{2}$ other districts' representatives by including spending in their districts in the proposal
representative of district $J(J \neq a)$ will vote for the bill if

$$
\begin{equation*}
H\left(g^{J}\right)-\tau \geq H\left(\bar{g}^{J}\right)-\bar{\tau} \tag{6}
\end{equation*}
$$

## the cost of buying votes

as long as $g^{J}<g^{J, D}$, then increasing $g^{J}$ raises the left side of (6)
not surprisingly, increasing spending in district $J$ makes the bill look more attractive to the representative from district $J$
but raising $g^{J}$ makes the bill less attractive to voters in every other district - and as long as $Z>3$, the agenda setter needs to get the vote of more than 1 other representatives
so the agenda setter does not want to spend more than is necessary

## a minimal coalition

because buying votes is costly, the agenda setter never spends more than necessary in other districts, so that

1. she has a proposal in which $g^{J}>0$ in exactly $\frac{Z-1}{2}$ other districts (besides her own)
2. if each district (other than a) for which $g^{J}>0$, the district's representative is just willing to vote for the proposal

$$
\begin{equation*}
H\left(g^{J}\right)-\tau=H\left(\bar{g}^{J}\right)-\bar{\tau} \tag{7}
\end{equation*}
$$

3. the districts for which $g^{J}$ are those for which vote buying is the cheapest : if $\hat{g}^{J}$ solves equation (7) for district $J$, then $g^{J}>0$ for the $\frac{z-1}{2}$ districts for which $N^{J} \hat{g}^{J}$ is smallest

## 3-District example

e.g. : 3 identical districts
$N^{J}=1 \quad J=1,2,3$
representative of district 1 gets to set the proposal default option : zero spending in each district
so district 1 needs support of 1 of the other 2 identical districts without loss of generality, make it district 2 whose support the member from district 1 seeks
(so $g_{3}=0$ )

## District 1's Maximization Problem

pick $g_{1}$ and $g_{2}$ so as to maximize

$$
\begin{equation*}
y-\frac{g_{1}+g_{2}}{3}+H\left(g_{1}\right) \tag{8}
\end{equation*}
$$

subject to getting district 2's approval

$$
\begin{equation*}
y-\frac{g_{1}+g_{2}}{3}+H\left(g_{2}\right) \geq y \tag{9}
\end{equation*}
$$

so the Lagrangean function is

$$
\begin{equation*}
y-\frac{g_{1}+g_{2}}{3}+H\left(g_{1}\right)+\lambda\left[-\frac{g_{1}+g_{2}}{3}+H\left(g_{2}\right)\right] \tag{10}
\end{equation*}
$$

## First-Order Conditions

$$
\begin{align*}
H^{\prime}\left(g_{1}\right) & =\frac{1+\lambda}{3}  \tag{11}\\
\lambda H^{\prime}\left(g_{2}\right) & =\frac{1+\lambda}{3} \tag{12}
\end{align*}
$$

characteristics of optimum

1. $\lambda<1$
2. over-supply in district 1
$g_{1}>g^{*}$
3. better to be in charge :
$W_{1}>W_{2} \quad g_{1}>g_{2}$
