Social Orderings

are (slightly) different from social choice rules

social choice rule : picks a single “best” alternative from the list of available alternatives v,w ,x ,y ,z ,. . ., based on voters’ rankings of the alternatives

social ordering : generates an ordered ranking, top to bottom, of the list of available alternatives v,w ,x ,y ,z ,. . ., based on voters’ rankings of the alternatives

so the social ordering must be (from its definition) **transitive**

Arrow's Theorem : Axioms

UD ("universal domain") : The rule we use to get social ordering, generated from individual voters' rankings, should work for **any** group of voters, **any** group of alternatives, and **any** profile of ranking of the alternatives by the voters.

P : ("Pareto principle") : If every single voter ranks alternative x above alternative y , then the rule must rank x above y in the social ordering. This is a very minimal way of requiring that the rule somehow pay attention to the actual rankings of the voters.

ND : ("Non–dictatorship") : The rule for a social ordering might give a lot of influence to person 1, for example. That's o.k. (at least under the weak requirements that Arrow proposed). But the rule can't be just "let's use person 1's ranking as the social ordering". Precisely : there is no single voter i, such that the social ordering is **always exactly** person i's ranking, no matter what are the rankings of the other voters.

IIA : ("Independence of Irrelevant Alternatives") : The relatively ranking of alternatives x and y in the social ordering must depend **only** on how the voters rank x and y , and not how voters rank x or y relatively to some "irrelevant" other alternative z .

Arrow's Impossibility Theorem

If there are at least 3 voters, and at least 3 alternatives, then there is no rule for generating a social ordering which satisfies all 4 axioms : UD, P, ND and IIA.

More About the Independence of Irrelevant Alternatives

profile *i*

	voter #1	voter #2	voter #3	voter #4	voter #5
first choice	w	x	z	y	y
second choice	v	v	v	v	z
third choice	x	y	w	z	v
fourth choice	y	w	x	x	w
fifth choice	z	z	y	w	x

profile *ii*

	voter #1	voter #2	voter #3	voter #4	voter #5
first choice	w	x	z	y	y
second choice	x	v	w	v	v
third choice	v	y	x	z	z
fourth choice	y	w	v	x	w
fifth choice	z	z	y	w	x

IIA says

if the x was above w in the social ordering for profile *i*, then x must be above w in the social ordering for profile *ii*, and ...

if the w was above x in the social ordering for profile *i*, then w must be above x in the social ordering for profile *ii*

in going from *i* to *ii* no voter changed the way she or he ranked x compared to w ; so the social ordering can't change the way it ranks x compared to w

IIA for Plurality (or Borda Count)?

profile *iii*

	voter #1	voter #2	voter #3
first choice	w	w	y
second choice	x	x	z
third choice	y	y	x
fourth choice	z	z	w

profile *iv*

	voter #1	voter #2	voter #3
first choice	x	x	y
second choice	w	w	z
third choice	y	y	x
fourth choice	z	z	w