# Distributive Spending

(cf. *Persson and Tabellini*, problem #6, page 114)

a fixed "pie" of size $r$ has to be divided among $2n+1$ districts

(where $n$ is any integer)

so a bill is just a vector of payments $(x_1, x_2, \ldots, x_{2n+1})$, with $x_i \geq 0$, and $\sum_{i=1}^{2n+1} x_i \leq r$, where $x_i$ is the amount of spending in district $i$

each district has a single representative, who cares only about the amount of spending in her own district

# no Condorcet winner

e.g. : $r = 2$, $n = 1$, then the "divide it equally proposal", $(x_1, x_2, x_3) = (2/3, 2/3, 2/3)$ will be defeated by the proposal

$(1, 1, 0)$, since representatives of districts #1 and #2 both prefer the second proposal

LEFT TO THE READER : (*i*) find a proposal which will defeat $(1, 1, 0)$ ; (*ii*) find another proposal which will defeat the proposal you found in part (*i*)

# an infinite agenda

in stage 1, one of the $2n + 1$ is chosen randomly to get to make a proposal $(x_1, x_2, \ldots, x_{2n+1})$ ; if the proposal passes, that's it

if the initial proposal is defeated in stage 1, then we go to stage 2 ; again a proposer is chosen at random and again the story ends if the allocation he proposes is passed

if the proposal in stage 2 is defeated, then we go to stage 3, in which a proposer is again chosen at random and so on

so the process ends as soon as a proposal is passed, but the process could go on forever

# minimum coalition

if a representative gets to make a proposal, she'll want the coalition to be the smallest possible winning coalition

that is, she'll want to "bribe" exactly $n$ other legislators with a bribe of $p$ dollars for each of their districts, in order to get the $n$ legislators (who are getting $p$ each) to vote in favour of her proposal, and the other $n$ legislators, who are getting nothing, to vote against

(of course she'll vote for her own proposal, so that "bribing" $n$ other legislators will get her $n + 1$ votes in favour of the proposal, which is the smallest number needed to pass a proposal)

# a stationary equilibrium

suppose that each legislator would decide to allocate $p$ dollars to each of the $n$ districts of the legislators whose support she is seeking

that is, each legislator has the same strategy, if she would happen to be chosen (at some stage) to propose : offer $p$ to $n$ other districts, and allocate the remainder, $P \equiv r - np$ to her own district

what is $p$, if ($i$) each legislator offered $p$ will agree to vote for the proposal ; ($ii$) $p$ is the smallest amount needed to get the legislator's support for the proposal?

# if the bill is defeated..

if this period's proposal is defeated, then in the next period, each legislator

has a chance of $\frac{1}{2n+1}$ of getting to make the next proposal

has a chance $\frac{n}{2n+1}$ of being offered $p$ to vote for the next proposal

has a chance $\frac{n}{2n+1}$ of being left out of the coalition voting in favour of the next proposal

# should she support the proposal?

in the **current** period, if one of the $n$ legislators being offered $p$ to vote for the proposal decides to vote against it, then the bill will be defeated

leaving her with an expected payoff of

$$\frac{1}{2n+1}P + \frac{n}{2n+1}p + \frac{n}{2n+1}0 = \frac{1}{2n+1}(r - np) + \frac{n}{2n+1}p$$

—or $\frac{r}{2n+1}$ in the next period

i.e. a share $\frac{1}{2n+1}$ of the available money $r$

if she discounts the future with discount rate $\beta$, then her expected payoff, if the current proposal is defeated, is

$$\frac{\beta r}{2n+1}$$

if the proposal passes, she'll get $p$

so the minimal payoff $p$ which will leave her willing to support the proposal is

$$p^* \equiv \frac{\beta r}{2n+1}$$

leaving the person making the proposal with

$$P^* \equiv r - np^* = r(1 - \frac{\beta n}{2n+1})$$