Proportional Income Tax Example

(similar to —but not the same as — example 1 [pg. 24] in *Persson and Tabellini*)

here : voters differ (only) in "ability" : how much they can earn per hour

and vote over a proportional income tax rate q

with tax revenue paid back to the taxpayers (an equal amount for everyone)

Utility

each voter's preferences can be represented by a utility function

$$c^i + V(x^i) \qquad (1)$$

where c^i is voter i's consumption (measured in dollars) and x^i is her leisure time (with $V'(\cdot) > 0$ and $V''(\cdot) < 0$

x_i is fraction of time spent at leisure, and l_i is fraction spent working

$$l^i + x^i = 1 \qquad (2)$$

(in *Persson and Tabellini*, people differ in the amount of time they have available)

Productivity Differences

let ω_i be person *i*'s productivity : how much she earns (before taxes) per period of time

(in *Persson and Tabellini*, this is the same for everyone)

if each person gets (the same) grant f from the government, then

$$c^i = (1 - q)\omega^i l^i + f \tag{3}$$

Government Budget Constraint

the grant f is financed by the proportional income tax, so that

$$f = q[\overline{\omega^i l^i}] \tag{4}$$

where $\overline{\omega^i l^i}$ is the average income : the total income of all the voters, divided by the number of voters

write the grant as $f(q)$, to show that the size of the grant depends on the tax rate through equation (4)

Choosing How Much to Work

for a given tax rate, a voter chooses her labour supply l^i to maximize her utility $c^i + V(x^i)$, subject to her budget constraint (3)

assume : person is a small enough part of the whole population, that she ignores the effect of her own labour supply on the average tax revenue collected

so she chooses l^i to maximize

$$f(q) + (1 - q)\omega^i l^i + V(1 - l^i) \tag{5}$$

First–Order Conditions

maximizing expression (5) with respect to l^i :

$$(1-q)\omega^i - V'(1-l^i) = 0 \tag{6}$$

as long as $V'' < 0$, then expression (6) means that

PROPOSITION 1: A person's labour supply l^i is an increasing function of her wage rate ω^i.

a person's labour supply can be written as a function $l^i(q, \omega^i)$, with l^i increasing in ω^i (and decreasing with q)

How The Tax Rate Matters for Person i

a person's utility, if the tax rate q is chosen, and if she choose a labour supply of l^i, will be

$$f(q) + (1-q)\omega^i l^i + V(1-l^i)$$

the derivative of this expression with respect to q is

$$\frac{dU^i}{dq} = f'(q) - \omega^i l^i + \frac{\partial l^i}{\partial q}[(1-q)\omega^i - V'(1-l^i)] \qquad (7)$$

using the condition (6) for her best choice of labour supply, equation (7) becomes

$$\frac{dU^i}{dq} = f'(q) - \omega^i l^i \qquad (8)$$

High–Wage People like Lower Tax Rates

from equation (8), and the fact (Proposition 1) that high–ω people work more hours,

PROPOSITION 2: Take a particular tax rate q. Then $\frac{dU^i}{dq} > \frac{dU^k}{dq}$ if and only if $\omega^i < \omega^k$.

so if some person i benefits from a tax increase, then so will all other people of lower ability than person i

Single Crossing

Take two different tax rates, q'' and q', with $q'' > q'$. Suppose that a person of ability ω^i prefers q'' to q'.

Then

PROPOSITION 3 : If $q'' > q'$ and $\omega^i > \omega^k$, and if person i prefers policy q'' to policy q', then person k must also prefer the higher–tax policy q'' to q'.

This is exactly the "single–crossing" property defined in Definition 3 (pg. 23) of *Persson and Tabellini*.