Pairwise Majority Rule

1 Pairwise?

Pairwise majority rule is the rule which most legislatures use to choose policies. It is used in Canada's House of Commons, provincial parliaments, the American Senate and House of Representatives, most countries' legislatures, and most cities' City Councils.

The key feature of this rule is that we always are voting over **pairs** of alternatives.

The pairs are : the current law, and some new, changed, law which some legislator has proposed.

So the provincial sales tax rate in Ontario is currently 8%. Any MPP[1] has the right to propose a bill in the provincial parliament. Some MPP could propose cutting the provincial sales tax to 7%. The members of the provincial parliament have to vote on this proposal : a vote of "yes", in favour of the proposal, is a vote in favour of a provincial sales tax of 7%, while a vote of "no" is a vote in favour of retaining the existing provincial sales tax of 8%.

So, if the current tax rate is 8%, and the proposal on the floor of the legislature is the proposal to cut the rate to 7%, legislators get to vote only for one of these two alternatives. If a majority of legislators prefer the new lower tax rate of 7% to the **status quo** of 8%, then the new proposal passes.

What if some legislator actually prefers a tax rate of 9%? That alternative is not available when legislators vote on the proposed law : the legislature's choices are "yes" (in favour of the 7% tax), "no" (in favour of the 8% tax), or "abstain".

But any legislator can propose a bill. So after the first bill has been voted on, this legislator now can proposed to raise the tax to 9%. This second bill will be put to a vote. Again, the vote on this second bill is a choice between a pair of alternatives : a "yes" vote is a vote in favour of the 9% tax rate, and a "no" vote is a vote in favour of whatever tax rate won the first vote (either 7% or 8%).

So only pairs of tax rates are ever compared. But any possible tax rate can get proposed (if some legislator likes that tax rate), so that eventually, a lot of pairs of alternatives will get compared.

[1] Member of the Provincial Parliament

2 Not the Same as Elections

We also use the term "majority rule" to describe how we run elections, for mayors, city councillors, and members of parliament.

But the procedure is not the same.

I will use the term "simple plurality" to describe the procedure used in most elections in Canada. This procedure is **not** pairwise. In voting for mayor, electors get a list of **all** the alternatives, and get to cast a vote for one of them. I will discuss this procedure, but not here. The social choice rule being analyzed in this lecture is pairwise majority rule, which is used mostly by legislatures and councils.

3 What's the Social Choice Rule?

Under pairwise majority rule, an alternative is called a **Condorcet[2] winner** if it defeats each other alternative in a pairwise vote.

That is, if there are 4 alternatives, w, x, y, and z, then the alternative y is a Condorcet winner if (and only if) at least half the voters prefer y to w, **and** at least half the voters prefer y to x, **and** at least half the voters prefer y to z.

So the social choice rule under pairwise majority rule is that the alternative chosen is the Condorcet winner.

Could there be more than one alternative which is a Condorcet winner? Yes, but only if two alternatives exactly tied in a vote between them, **and** those two alternatives beat (or tie with) every other alternative in a pairwise vote. In such a case, we would have a tie for our winner. That's not a problem, but it's also not a very interesting case. And it can't happen if we have an odd number of voters, if each voter has a strict ranking (no ties) of all the alternatives.

4 The Condorcet Paradox

The problem with using pairwise majority rule as a social choice rule is not that there are too many winners : the problem is that there may be no Condorcet winner at all. The simplest example of a situation in which this problem arises has 3 voters and 3 alternatives. This example is called the Condorcet Paradox. and is shown in the table below.

	voter #1	voter #2	voter #3
first choice	x	y	z
second choice	y	z	x
third choice	z	x	y

[2]Condorcet was a philosopher/mathematician who lived in France in the late eighteenth century, and who was one of the pioneers of the analysis of social choice.

What happens in a pairwise vote between alternatives x and y? The alternative x wins by a 2–to–1 vote : voters #1 and #3 both rank x above y, and so would vote for x if the other alternative is y.

But to be a Condorcet winner, x must defeat every other alternative in a pairwise vote. And her x will lose to z in a pairwise vote : both voter #2 and voter #3 rank z above alternative x, so that z defeats x by a vote of 2–to–1 in a pairwise contest.

And z is not a Condorcet winner either. In a pairwise vote against y, z loses by a 2–to–1 vote, since voter #1 and voter #2 both rank y above z.

And y can't be a Condorcet winner, since the first vote had it losing (by a 2–to–1 majority) to alternative x.

5 No Social Ordering

The Condorcet Paradox also shows that we can't use pairwise majority rule to generate a transitive ordering of all the alternatives. Here x beats y which beats z which beats x. Even though each of the 3 voters has a transitive ordering of the 3 alternatives, pairwise majority rule does not generate an overall social ordering.

6 A Sneaky Way Out?

Since ties are o.k., why don't we just say everything is tied, when we don't have a Condorcet winner?

That won't be a very decisive rule, of course.

But it also won't even resolve the paradox. Suppose there are four alternatives, and three voters, with the following profile of preferences :

	voter #1	voter #2	voter #3
first choice	x	y	w
second choice	y	w	z
third choice	w	z	x
fourth choice	z	x	y

In this example, there is still no Condorcet winner.[3] But it does not make sense to rate all 4 alternatives as "just as good as each other", since here all 3 voters agree unanimously that w is strictly better than z.

[3] z still beats x, x still beats y, and y still beats z. And the new policy w isn't a Condorcet winner either, since it loses a pairwise election to y.

7 Supermajorities to the Rescue

Sometimes legislation needs more than a simple majority to pass. For example, in some cases a new proposal may require the support of 2/3 of the voters, rather than a simple majority, in order to defeat the status quo.[4]

There are several arguments made in favour of requiring supermajorities, such as 2/3, which are greater than 50%. Most of these arguments are wrong.

One argument is that supermajorities help preserve the rights of minorities. That may be true, but only if the status quo is good to the minorities. Requiring a supermajority of men to approve, in order to change the rules to allow women to vote — as was the case in the United States — doesn't seem to help the rights of females[5].

Another argument is that supermajorities create more stability, preventing the cycles that arise in the Condorcet paradox. This argument is also wrong. When there are three voters, a simple majority is the same thing as a two–thirds majority. If we required a 2/3 majority for a proposal to be ranked above another, nothing would change in the two examples in the tables in this lecture : in each case each proposal can be defeated by another, even if the other proposal requires 2/3 of the votes.

[4]In the U.S. Congress, for example, a proposal which has been vetoed by the President can only be passed if it gets 2/3 of the votes.

[5]who technically are a majority, not a minority