Probabilistic Voting

(cf. *Persson and Tabellini*, 3.4)

policy space is still 1–dimensional — what level g of per capita expenditure to provide

still 2 parties, trying to maximize probability of getting elected, still committing to policies g^A and g^B

voters still differ in their income y^J

What's New : I

but now the parties differ in their popularity, measured by δ

$\delta > 0$ means that (all) people like party *B* better ; the bigger is δ, the bigger the popularity advantage for party *B*

a voter's utility when the public expenditure is

$$W^J(g) \equiv (\bar{y} - g)\frac{y^J}{\bar{y}} + H(g)$$

(as in the "minimum differentiation" model), but a person of income y^J will vote for party *A* over party *B* if and only if

$$W^J(g^A) > W^J(g^B) + \delta$$

this popularity measure δ is the same for everyone

and it can vary

assumption : δ is a random variable, drawn from the uniform distribution over $[-\frac{1}{2\psi}, \frac{1}{2\psi}]$

so δ can take any value between $-\frac{1}{2\psi}$ and $\frac{1}{2\psi}$, and every value in this interval is equally likely

parties know what ψ is, but they don't know the actual value of δ when they make their policy choices

if δ is big and positive, only votes for party A are from people for whom $W^J(g^A)$ is a lot bigger than $W^J(g^B)$ (for whom $W^J(g^A) - W^J(g^B) > \delta$)

What's New *II*

the popularity parameter δ is the same for everyone

but there's a second new element, an "idiosyncratic" bias among voters (for one party or the other)
which differs among people

so there are many voters of income y^J ; they also vary in their personal preference σ^{iJ}

voter *iJ*'s overall preference for party *B* over party *A* is $\sigma^{iJ} + \delta$
so she'll vote for party *A* only if

$$W^J(g^A) > W^J(g^B) + \delta + \sigma^{iJ}$$

for each income level y^J, these biases σ^{iJ} are uniformly distributed over some interval

$$[-\frac{1}{2\phi^J}, \frac{1}{2\phi^J}]$$

parties know about these biases ; each party knows, for example, that $1/4$ of all the voters of income y^J have a bias in favour of party B of $\frac{1}{4\phi^J}$ or more

The Swing Voter

the voter of personal bias σ^J is defined as the voter (of income y^J) who is indifferent between the parties :

$$W^J(g^A) = W^J(g^B) + \sigma^J + \delta \quad (1)$$

or

$$\sigma^J = W^J(g^A) - W^J(g^B) - \delta \quad (2)$$

everyone whose bias is less than σ^J votes for party A

that's a fraction

$$\frac{1}{2} + \sigma^J \phi^J \quad (3)$$

The Overall Vote

if a fraction α^J of the voters have an income y^J (and these voters vary in their biases), and party *A* gets a share $\frac{1}{2} + \sigma^J \phi^J$ of those voters' votes, then equation (2) implies that party *A*'s overall vote is

$$\frac{1}{2} + \sum_J \alpha^J \phi^J [W^J(g^A) - W^J(g^B) - \delta] \tag{4}$$

party *A* wins if this share is greater than $\frac{1}{2}$, which will happen if

$$\sum_J \alpha^J \phi^J [W^J(g^A) - W^J(g^B)] > \sum_J \alpha^J \phi^J \delta \tag{5}$$

Probability of Winning

the probability that the popularity parameter δ is less than x is

$$Prob(\delta < x) = \frac{1}{2} + \psi x \tag{6}$$

so that equation (5) says that party A's probability of winning is

$$\frac{1}{2} + \frac{\psi}{\phi}(\sum_J \alpha^J \phi^J [W^J(g^A) - W^J(g^B)]) \tag{7}$$

where ϕ is the average value of the ϕ^J's :

$$\phi \equiv \sum^J \alpha^J \phi^J$$

Part A's Platform

party A wants to maximize its chance of winning ; so it should choose a policy g^A to maximize expression (7)

taking as given the policy g^B chosen by its rival

so g^A is chosen so that

$$\sum_J \alpha^J \phi^J \frac{dW^J}{dg^A} = 0 \tag{8}$$

What About Party *B*?

party *B* wants to maximize its own chance of winning, given party *A*'s policy g^B
so that it chooses g^B so as to maximize

$$\frac{1}{2} + \frac{\psi}{\phi}(\sum_J \alpha^J \phi^J [W^J(g^B) - W^J(g^A)]) \tag{9}$$

so that

$$\sum_J \alpha^J \phi^J \frac{dW^J}{dg^B} = 0 \tag{10}$$

as in the simple (no uncertainty) Hotelling–Black–Downs model, parties here choose the same policies in equilibrium

Equilibrium Policy

the policy each party chooses — the solution to (8) [or (10)]

maximizes a weighted sum of different groups' interests

this solution is the g which maximizes

$$\sum_J \alpha_J \phi^J W^J(g) \tag{11}$$

conclusion : more weight on groups with high ϕ^j — which means "less–spread–out" distribution of the indiosyncratic characteristic σ^{iJ}

high ϕ^J means more responsive to small changes in policy, which means politicians pay more attention to such groups